Approximations of distributions for some standardized partial sums in sequential analysis

Abstract In sequential analysis it is often required to determine the distribution of $\sqrt{t}\overline{Y}_t$ and/or $\sqrt{a}\overline{Y}_t$, where t is a stopping time of the form $t = \inf\{n \geq 1 : n + S_n + \xi_n > a\}$, $\overline{Y}_n$ is the mean of n i.i.d. random variables Y_i with mean zero and variance one, S_n is the partial sum of i.i.d. random variables X_i with mean zero and a positive finite variance, and $\{\xi_n\}$ is a sequence of random variables that converges in distribution to a random variable ξ as $n \to \infty$ and ξ_n is independent of $(X_{n+1}, Y_{n+1}), (X_{n+2}, Y_{n+2}), \cdots$ for all $n \geq 1$. Anscombe's (1952) central limit theorem asserts that both $\sqrt{t}\overline{Y}_t$ and $\sqrt{a}\overline{Y}_t$ are asymptotically normal for large a, but a normal approximation is not accurate enough for many applications. Refined approximations are available only for a few special cases of the general setting above and are often very complex. In this paper some simple Edgeworth approximations are provided. They are numerically satisfactory for the problems considered in this paper.