On Testing Homoscedasticity via Independence Under Joint Normality

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On the basis of a random sample of n observations from a p-variate normal distribution, with unknown parameters, we want to test the homoscedasticity hypothesis that the p variances are equal. The likelihood ratio criterion for the bivariate case $X = (X_1, X_2)$ turned out to be equivalent to testing the independence of $X = X_1 + X_2$ and $Y = X_1 - X_2$ (Morgan, *Biometrica* 1939). Hence the corresponding well-known statistics t has the t_{n-2} distribution, with $n - 2$ degrees of freedom. Here it is shown that, under a general covariance structure, testing homoscedasticity for $p > 2$ is not reducible to testing independence of linear combinations of the X_i's. However, if p is equal to $2k$, then the analogous sum difference transformation can be used to test the hypothesis that the covariance matrices of $X^{(1)} = (X_1, \cdots, X_k)$ and $X^{(2)} - (X_{k+1}, \cdots, X_{2k})$ are the same, provided that the covariance matrix of $X^{(1)}$ and $X^{(2)}$ is symmetric. Then, the distribution of the corresponding likelihood ratio criterion for testing the independence of $X^{(1)} + X^{(2)}$ and $X^{(1)} - X^{(2)}$ is known (Anderson 1958, 1984). It is noted that, whereas when X_1 and X_2 are independent, the F-test is based on an $F_{n-1,n-1}$ distribution, or, equivalently, on a t_{n-1} distribution, in view of the relation $t_n = n^{1/2}/2(F_{n,n}^{1/2} - F_{n,n}^{-1/2})$ (Cacoullos, *JASA* 1965), in the above bivariate case of correlated X_1 and X_2 the t-statistic is on $n - 2$ degrees, a loss of one degree of freedom, due to the unknown correlation. Two special covariance matrix structures are also considered in the general case $p > 2$.