On my honor, I have neither given nor receive unauthorized aid on this examination.

Signature______________________________
1. Suppose $X$ and $Y$ are random variables with joint probability density function given by

$$f(x, y) = \begin{cases} \frac{1}{2}xy, & 0 \leq x \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

A. Find the marginal distribution of $Y$.

B. Find the conditional distribution of $X$ given $Y = y$.

C. Are $X$ and $Y$ independent? Justify your answer.
2. A soft-drink machine has a random amount $Y$ in supply at the beginning of a given day and dispenses a random amount $X$ during the day (with measurements in gallons). It is not resupplied during the day and hence $X \leq Y$. It has been observed that $X$ and $Y$ have the joint probability density function given in problem (1). (You may use the work you did in problem (1) to help answer the questions below if you desire to do so.)

A. Evaluate the probability that less than 1 gallon is sold given that the machine contains 2 gallons at the start of the day.

B. Find the expected number of gallons sold given that the machine contains 2 gallons at the start of the day.
3. For a particular large orchard, the number of apples $N$ on an apple tree is distributed as a Poisson random variable with a mean of 50. Each apple has the probability 0.05 of being damaged, and this event is independent from apple to apple.

A. Find the mean number of damaged apples per tree.

B. Find the variance of the number of damaged apples per tree.
4. An automobile dealer has three models available, the “economy,” the “mid-size,” and the “full-size.” A salesman working for the dealer estimates that, for a given customer entering the showroom, the probability of no sale is 0.7, of selling an economy is 0.15, of selling a mid-size is 0.10, and of selling a full-size is 0.05. Of course, he makes no commission if there is no sale. His commission on the economy, mid-size, and full-size is $200, $400, and $600, respectively. Assume the next six customers act independently of one another. (No customer will purchase more than one car.)

A. What is the probability that the salesman will sell 2 economy and 1 full-size car to the next six customers?

B. What is the expected commission that the salesman will earn from the next six customers?
5. Let $X$ be an exponential random variable with a mean of 1. Define $Y = \ln(X)$.

A. Find the distribution function of $Y$.

B. Find the probability density function of $Y$. 