Problem 2.18

1. Proof:

\[
P(Y < y) = P(U < y | U < \alpha) = \frac{P(U < y, U < \alpha)}{P(U < \alpha)}
= \begin{cases} 
  \frac{P(U < y)}{\alpha} & \text{if } y < \alpha \\
  \frac{P(U < \alpha)}{\alpha} & \text{if } \alpha < y 
\end{cases}
= \frac{y}{\alpha} I_{(0, \alpha)}(y) + I_{(\alpha, \infty)}(y)
\]

This is the c.d.f. of a \( U(0, \alpha) \).

2. Based on the histograms on page 2, there does not seem to be much difference between the two algorithms in generating random numbers, even when \( \alpha \) is close to 1.

3. R-code for algorithm A.14:

```r
for(a in c(0.8, 0.9, 0.99))
{
  n <- 12000 s <- 10000
  x <- runif(n)
  x <- x[x < a]
  x <- x[1:s]
  y <- a*runif(s)
  hist(x, main=paste(a), breaks=seq(0,a,length=30), freq=F)
  hist(y, main=paste(a), breaks=seq(0,a,length=30), freq=F)
}
```
Figure 1: Histograms of 10,000 $\text{Uniform}(0,\alpha)$ generated with algorithm A.14 (left column) and the genuine transformation $\alpha^* U(0,1)$ (right column).