Monte Carlo Statistical Methods

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Problem 3.10

We know that

\[
\frac{1}{M} \sum_{m=1}^{M} h(X_m) \rightarrow E(h(X))
\]

We have to show:

\[
E(h(X)) = \frac{\sum_{i=1}^{M} h(y_i) f(y_i) g(y_i)}{\sum_{j=1}^{M} f(y_j) g(y_j)}
\]

By the given algorithm, we know that

\( X_m = Y_i \) with prob \( w_i \) for all \( m=1,..., M \)

Now,

\[
LHS = E(h(X))
= \sum_{i=1}^{M} h(y_i) \times w_i
= \frac{\sum_{i=1}^{M} h(y_i) f(y_i)}{\sum_{j=1}^{M} f(y_j)}
= RHS
\]