STA6326
Prof. Casella

Assignment 3
Due Wednesday September 14

(1). Suppose that $E$ and $F$ are mutually exclusive events of an experiment. Show that if independent trials of this experiment are performed, then $E$ will occur before $F$ with probability $P(E) / [P(E) + P(F)]$

(2). Starting from the assumption that $P(E_1 \cap E_2) \geq P(E_1) + P(E_2) - 1$, use induction to establish Bonferroni’s inequality

$$P(E_1 \cap E_2 \cap \ldots \cap E_n) \geq P(E_1) + \ldots + P(E_n) - (n - 1)$$

(3). Disease screening is a topic that often provokes debates, but an analysis of probabilities can show the consequences of different policy decisions. The last debate (that I know of) occurred in the 1990’s concerning the wisdom of widespread testing for AIDS. Define the events $D = \{\text{person has AIDS}\}$ and $E = \{\text{AIDS test is positive}\}$. Assume $P(E|D) = .95$ and $P(E|D^c) = .001$ (think about what these probabilities mean).

Some excerpts from an article in the *Boston Globe*:

The conclusion of a federal study that looked at the pattern of HIV infections in 20 hospitals concluded that routine, voluntary testing of hospitalized patients between the ages of 15 and 54 could identify more than 100,000 individual every year who are unknowingly infected with the AIDS virus.

The study, conducted by scientists at the CDC, addressed criticism of proposed broad-scale testing as inefficient by focusing on a population more likely to be infected. The CDC group tested anonymous blood samples from hospitals and estimated that about 225,000 HIV-positive persons were admitted, but only one-third were admitted for problems related to AIDS. They also concluded that narrowing the testing to patients between the ages of 15 to 54 could identify 68 percent of unknowingly HIV-positive patients.

(a) Illustrate the inefficiency of broad scale testing by calculating the probability of a false positive, $P(D^c|E)$, in a population where the incidence of AIDS is 1 in 10,000
(b) The CDC recommendation is to focus on a population with higher AIDS incidence, say 1 in a 100. Show the change in false positive probability.

(c) Write the event “identify unknowingly HIV-positive patients” in terms of $D$ and $E$

(d) Give a selection of values of $P(E|D)$, $P(E|D^c)$, and incidence rates that would result in the identification of at least “68 percent of unknowingly HIV-positive patients”.

(4). From the book: 1.19, 1.26, 1.31, 1.36, 1.38, 1.41, 1.43, 1.44