(1). Sarmanov (1966, Soviet Mathematics) proposed the following form for a bivariate distribution:

\[ f(x, y) = f_1(x)f_2(y)[1 + \omega g_1(x)g_2(y)], \]

where \( f_i \) are density functions and \( g_i \) are bounded nonconstant functions satisfying

\[ 1 + \omega g_1(x)g_2(y) > 0 \quad \text{and} \quad \int g_i(t)f_i(t)dt = 0, \quad i = 1, 2. \]

(a) Show that \( f(x, y) \) is a joint density function.
(b) Show that \( f_1(x) \) and \( f_2(y) \) are the marginal distributions
(c) Let \( f_i \) be \( N(0,1) \) and let \( g_i(t) = \sin(t) \).
   i. Show that this is a legitimate joint density.
   ii. Find the conditional density of \( X \) given \( Y \), and graph the function
   iii. Show that \( \text{Cov}(X, Y) = \omega \).
(d) For \( f_i = N(0,1) \), find another function \( g_i(t) \) that gives a legitimate joint density.

(2). From the Book: 4.13, 4.20, 4.26, 4.33, 4.39, 4.41, 4.42, 4.44