These problems are intended as review problems; they are to help you brush up on some of the calculus you may have forgotten. If you are a little rusty, these problems may take you a bit longer than usual to complete, but it will be time well spent because we need these concepts and techniques throughout the course.

(1). Evaluate the following limits

(a) \( \lim_{x \to 2} \frac{2x^2 - 6}{x^2 + 1} \)
(b) \( \lim_{x \to \infty} \frac{x^2 - 2x + 3}{2x^2 + 5x - 1} \)
(c) \( \lim_{x \to 0} x^n \log a, \quad a > 0 \)
(d) \( \lim_{x \to 0} \frac{\log x}{\log(\sin x)} \)

(2). Differentiate

(a) \( 5(x^3 - 3x + 4)^6 \)
(b) \( 3\tan^2(2x) \)

(3). Sketch to a reasonable degree of accuracy:

(a) \( y = x + |x| \)
(b) \( y = (1 - x^2)^2 \)

Be sure all extrema are clearly indicated

(4). Integrate:

(a) \( \int_0^1 (1 - x)^n dx, \quad n > -1 \)
(b) \( \int_{\pi/2}^{\pi/6} \sin^3 \theta d\theta \)
(c) \( \int_0^\infty x^2 e^{-x} dx \) (Try integration by parts)

(5). Find the area

(a) between one arc of the curve \( y = A \sin \omega x \) and the \( x \)-axis where \( A \) and \( \omega \) are positive constants;
(b) inside the curve \( y^2 = x^2 - x^4 \);
(c) inside the curve \((x^2 + y^2)^2 = 2a^2xy, a \) constant. (This curve is called a \textit{lemniscate} for reasons totally unknown to me.) Try to draw a picture of it. Note that transforming to polar coordinates will simplify the integration.

(6). For each of the following series, decide if it converges or diverges

(a) \( \sum_{k=1}^{\infty} \frac{1}{k^{2k}} \)
(b) \( \sum_{k=1}^{\infty} \frac{1}{k^{1/3}} \)
(c) \( \sum_{k=2}^{\infty} \frac{1}{k \log k} \)

(7). Sum the following series

(a) \( \sum_{i=1}^{n} i^2 \)
(b) \( \sum_{k=0}^{\infty} p^k, \ |p| < 1 \)
(c) \( \sum_{k=0}^{\infty} kp^{k-1}, \ |p| < 1 \)
(d) \( \sum_{k=0}^{\infty} \frac{a^k}{k!}, \ a \) an arbitrary constant