Course Syllabus **Spring 2013**

STA 6934 Topics in Basic Analysis

CLASS: MWF Period 8 (3:00 p.m.- 3:50 p.m.) FLO 230

INSTRUCTOR:
Dr. Andrew Rosalsky
206 Griffin-Floyd Hall
Phone: 273-2983
e-mail: rosalsky@stat.ufl.edu

OFFICE HOURS: Period 6 (MW) and Period 7 (MWF)

OBJECTIVE: The objectives of this course are:
(i) to provide prospective statistics doctoral students with the *knowledge* of basic mathematical analysis which is needed for the study of measure-theoretic probability (STA 7466-7467), and
(ii) to enable prospective statistics doctoral students to reach a level of *mathmatical sophistication* needed for the study of measure-theoretic probability (STA 7466-7467).

Although this course does count as a masters elective, it is recommended for terminal masters students only if either they anticipate doctoral work in statistics sometime ahead in the future or they have a genuine interest in learning basic mathematical analysis as a component of their overall education. In general but not always, terminal masters students should register instead for a statistics elective course of an applied nature.

We will probably not be able to cover every topic from mathematical analysis which ideally should be known prior to the study of measure-theoretic probability. However, by completing this course, students should have developed the mathematical maturity to enable them to read on their own about any needed topic which was not covered.

It would *not* be appropriate to think of this course as "Calculus Four" or "Advanced Calculus". Indeed, it would be better to think of it as "Calculus Zero" or "Calculus Infinity" since we will be mainly concerned with foundational matters and the underlying theory of calculus rather than with developing a collection of methods and techniques. Precise definitions and theorems (and their proofs) form the main components of the course. Examples and counterexamples are also provided to elucidate the fine and subtle points of the general theory. Emphasis is placed on rigorous development of mathematical concepts and the development of critical thinking skills.

PREREQUISITES: As a minimum, three semesters of honest calculus are needed. Additional work in mathematics would of course be desirable but strictly speaking not necessary.

COURSE MATERIAL:
(i) STA6934 Course Notes (available from Target Copy on University Avenue)
(ii) Kenneth A. Ross, *Elementary Analysis: The Theory of Calculus.* Springer, New York (This book is optional.)

We will follow the Course Notes very carefully. The Ross text will only serve as a general reference. We will not cover every topic in the Ross text and we will also cover some topics which are not in the Ross text. As a rough guide, we will attempt to cover much of Chapters 1 to 4 and touch on parts of Chapters 5 and 6 of the Ross text. However, while the treatment of many of the topics covered will be identical to that in the Ross text except for minor modifications, the treatment of other topics covered will be substantially different from that given in the Ross text.

It may be mentioned that by far the most famous textbook containing the material presented in this course is the textbook: Walter Rudin, *Principles of Mathematical Analysis* , 3^{rd} edition. McGraw Hill, New York. This textbook is designed for a two semester course sequence. It has remained a top seller ever since the first edition appeared over 55 years ago! The reasons that the Ross text was chosen as the general reference rather than the Rudin text are that (i) the Ross text is basically intended for a one semester course, and (ii) the Ross text is far more "student friendly" than the Rudin text.

TOPICS: (Chapter numbers refer to the STA 6934 Course Notes)

Chapter 1. Review of set theory, the well-ordering property of the natural numbers and the principle of mathematical induction, the real numbers $\mathbf{R}$ with emphasis on the completeness axiom, supremum and infimum of a subset of $\mathbf{R}$, the Archimedean Property of $\mathbf{R}$.

Chapter 2. Sequences in $\mathbf{R}$, convergence, infinite limits, montone sequences, limit superior and limit inferior of a sequence and their relation to the limit of a sequence, Cauchy sequences, subsequences, the Bolzano-Weierstrass theorem, metric space topology with emphasis on Euclidean k spaces, open and closed sets, accumulation points, compactness, the Heine-Borel theorem, infinite series, alternating series, rearrangement of series.

Chapter 3. Limits of functions, continuity of functions, properties of continous functions, uniform continuity.

Chapter 4. Sequences and series of functions, convergence and uniform convergence of sequences and series of functions.

Chapter 5. Selected topics from differential calculus.

Chapter 6. Selected topics from the theory of the Riemann integral, definition of the Riemann-Stieltjes integral.

The above topics contain results obtained by some of the 19^{th} century's greatest mathematicians. Measure-theoretic probability (which was developed in the 20^{th} century) could not have been developed without the above topics. We acknowledge that it might not be transparent at the present time that knowledge and genuine understanding of the above topics are a basic and crucial component of a statistics doctoral program. This will be transparent next year in measure-theoretic probability (STA 7466-7467) as well as in some other advanced statistics courses.

It may be surprising that not much time will be spent on Chapter 5 (Differentiation) and Chapter 6 (Integration). Experience has shown that prior to the beginning of this course, students are already rather proficient with differentiation and (Riemann) integration concepts but have a superficial understanding of the foundational material in Chapters 1 to 4. Thus we emphasize the material in Chapters 1 to 4. Moreover, in STA 7466 we will develop and use another form of integration (the Lebesgue integral) which has nicer properties than the Riemann integral of calculus.

POLICIES: Course grades will be determined by performance on six exams, each worth 50 points.

Exam 1	(in class, closed book and notes) Chapter 1 and Chapter 2 (up to page 48 of Course Notes)
Exam 2	(take-home, you may refer to your Course Notes and the Ross text only) Chapter 1 and Chapter 2 (up to page 48 of Course Notes)
Exam 3	(in class, closed book and notes) Chapter 2 (starting with page 49 of Course Notes and Chapter 3)
Exam 4	(take-home, you may refer to your Course Notes and the Ross text only) Chapter 2 starting with page 49 of Course Notes and Chapter 3)
Exam 5	(in class, closed book and notes) Entire course
Exam 6	(take-home, you may refer to your Course Notes and the Ross text only) Entire course

The "in class" exams (Exams 1, 3, and 5) will consist of giving precise definitions, stating theorems, solving some "not too difficult" problems (proofs required) and providing examples/counterexamples. The "take-home" exams (Exams 2, 4, and 6) will consist of solving problems (proofs required) and providing examples/counterexamples. The "take-home" exam questions might be more difficult than the "in class" exam questions, or they might be of the same level of difficulty but take longer to write out the solution.

Exams 1 and 2 will occur at approximately the same time; similarly for Exams 3 and 4. Exam 5 will be on Wednesday April 24, 2013. Exam 6 will be handed out on Wednesday April 24, 2013 and is due on Friday April 26, 2013 by 3:00pm. (Take it to the Instructor's office anytime between 1:00pm and 3:00pm on Friday April 26, 2013.)

ATTENDANCE: Classroom attendance is fully expected.

ACADEMIC HONESTY : University of Florida students are expected to abide by the following: "We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity."

CLASSROOM ACCOMMODATIONS: Students requesting classroom accommodations must first register with the Dean of Students Office. The Dean of Students Office will provide the student with a letter to be given to the instructor when requesting accommodation.