STA 7466 **Probability Theory 1 (Section 5956)** **Fall 2012**

INSTRUCTOR:
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OFFICE HOURS: Monday, Wednesday, and Friday: Period 8

COURSE REQUIREMENTS:

Exam 1 (in class: 50 points)
Exam 2 (take-home: 50 points)
Exam 3 (in class: 50 points)
Exam 4 (take-home: 50 points)
Exam 5 (in class: 50 points)
Exam 6 (take-home: 50 points)

Exams 1 and 2 will be more or less at the same time; so will Exams 3 and 4. Exam 5 will be on Wednesday, December 5, 2012. Exam 6 will be handed out that same day and is due on Friday, December 7, 2012 between 2:00pm and 4:00pm. (Bring it to the Instructor's Office.) There will also be homework assignments. Exam questions will consist of original problems (proofs required) as well as questions calling for the construction of examples or counterexamples elucidating the fine points or subtleties of the theory (proofs required).

REQUIRED COURSE MATERIAL: "Notes on Measure, Integration, and Probability Theory" (revised Summer 2012) by Andrew Rosalsky. This is available (or soon will be available) at Target Copy on University Avenue.

PREREQUISITES: MAA 5228 or STA 6934 (Topics in Basic Analysis) with a grade of B or better. (The grade of B- does not satisfy the prerequisite.) Alternatively, the student may have taken an equivalent course at another institution. But such students must have achieved a passing score on the Department of Statistics Basic Analysis Exam given shortly before Fall Semester classes begin.

The student is assumed to have experience with and working knowledge of the following topics from elementary real analysis:

1. Set theory (e.g., set operations, countable unions of countable sets are countable).
2. Properties of a function from an arbitrary set A to an arbitrary set B (e.g., direct and inverse images and their relation to set operations).
3. The real number system (e.g., the rationals are countable, the irrationals are uncountable, the completeness property (or axiom) and the supremum of a set of real numbers).
4. The topology of the real number line (e.g., open sets, closed sets, compact or closed and bounded sets, limit points of sets).
5. The epsilon-delta approach to limit and convergence concepts (e.g., Cauchy sequences, subsequences, upper and lower limits, uniform convergence of functions).
6. Continuity and uniform continuity (e.g., a continuous function on a closed and bounded interval is uniformly continuous).
7. Riemann-Stieltjes (or at least Riemann) integration.

The student familiar with most of the topics in elementary books such as Rudin's Principles of Mathematical Analysis (first 7 chapters) or Ross's Elementary Analysis: The Theory of Calculus is well prepared to enter this course. Although it is recognized that all students have had experience with elementary probability, no official background in probability is presupposed.

COURSE OBJECTIVE: The two-semester sequence STA 6466–6467 covers that material from measure, integration, and probability theory that every statistics doctoral student should know. Since the axiomatic approach to probability theory (which was developed by the eminent Russian probabilist Andrei N. Kolmogorov and published in Germany in 1933 under the title Grundbegriffe der Wahrscheinlichkeitsrechnung) rests on the theory of abstract measure and integration, about 12 weeks of the first semester is devoted mainly to this theory. Many of these topics from measure and integration theory are those included in a standard graduate course generally titled "The Theory of Functions of a Real Variable". However, more emphasis will be placed on the study of probability spaces, random variables, and distribution functions than is generally found in a real variables course. The modern approach to measure and integration was initially developed by the renowned French mathematician Henri Lebesgue. Lebesgue published his work at Paris in 1903 under the title Leçons sur L'Intégration et la Recherche des Fonctions Professées au Collège de France. Since that time the Lebesgue theory has been generalized. Specifically, the abstract Lebesgue integral was introduced by the great French probabilist Maurice Fréchet in 1915 (Sur l'intégrale d'une fonctionelle étendue à un ensemble abstrait, Bull. Soc. Math. France 43, 248-265). In STA 6467, we will apply our knowledge of measure and integration theory to the study of the interesting, important, and useful topics such as independence, modes of convergence of random variables, convergence of series of independent random variables, weak and strong laws of large numbers, characteristic functions, the central limit problem, stopping times.

COURSE INTRODUCTION: The axiomatic approach to probability theory (as developed by Kolmogorov) postulates:

(i) A set Ω of fundamental occurrences (Ω is called the sample space and is the abstract counterpart of the collection of primitive outcomes (sample points ω) of a real world experiment),

(ii) A set $\mathcal{F}$ (of subsets of Ω) of so-called (chance) events,

(iii) A set function $P(\cdot)$ on $\mathcal{F}$ called the probability of the event constituting its argument ($P(\cdot)$ is the abstract counterpart of long-term relative frequency) and stipulates that:

- (a) $\mathcal{F}$ is a σ-algebra of subsets of Ω,
- (b) P is a measure,
- (c) $P(\Omega) = 1$.

It follows that $P(\emptyset) = 0$, $0 \le P(A) \le 1$ for each A in $\mathcal{F}$, and moreover $P(\lim A_n) = \lim P(A_n)$ for any sequence of events $\{A_n, n \ge 1\}$ whose limit exists. Such a triplet $(\Omega, \mathcal{F}, \mathcal{P})$ is called a probability space and is a particular example of a finite measure space.

A random variable X is an $\mathcal{F}$-measurable transformation from Ω to the Borel line with $|X| < \infty$ almost certainly (a.c.). The expected value of X is defined to be the (abstract) Lebesgue integral of X over Ω with respect to P

$$EX = \int_\Omega X(\omega) dP(\omega) \left(= \int_\Omega X^+(\omega) dP(\omega) - \int_\Omega X^-(\omega) dP(\omega) \right)$$

provided the integral exists (the values $+\infty$, $-\infty$ are admissible values of EX). If $F(x) = P(X < x)$ is the distribution function of X, then

$$EX = \int_\Omega X(\omega) dP(\omega) = \int_R x \; dF(x) \text{ (Lebesgue-Stieltjes integral)}$$

in the sense that if either integral exists, then so does the other and they are equal. If EX exists and is finite, then X is said to be integrable.

The student is not, of course, expected to have an understanding of this introduction until after about the 10th week of STA 6466. Nevertheless, the concepts outlined here form the foundation of the theory of probability.

LIST OF TOPICS:

1. Review of set theory, set operations, indicator functions.
2. Classes of sets, sigma algebras, the lambda-pi class theorem, measurable spaces, product spaces, Borel sets.
3. Measurable transformations, Borel functions, the monotone system theorem.
4. Set functions, measures, probability spaces, induced measures, distribution functions.
5. Convergence almost certainly, convergence in probability, Egorov's theorem, convergence almost everywhere, convergence in measure.
6. (Abstract) Lebesgue integration over a measure space and over a probability space (expected value), properties of the integral, Markov inequality, Lebesgue monotone convergence theorem, indefinite integrals, the Fatou lemma, Lebesgue dominated convergence theorem, $\mathcal{L}_p$ spaces, convergence in $\mathcal{L}_p$, relating absolute moments to the tail of a distribution (large deviation results).
7. Uniform integrability on a probability space, Cauchy convergence criterion (Reisz-Fischer completeness of $\mathcal{L}_p$), mean convergence criterion.
8. The Jensen, Liapounov, Hölder, Schwarz, and Minkowski inequalities.
9. Measure extension, semi-algebras, outer measures, Carathéodory extension theorem, complete measure spaces, monotone functions, distribution functions, Lebesgue-Stieltjes measures, Lebesgue measure, the "construction" of a non-Lebesgue measurable set.
10. Lebesgue-Stieltjes and Lebesgue integrals, change of variable theorem, relations between Lebesgue-Stieltjes and Riemann-Stieltjes integrals, the Cantor ternary set and function.
11. Product measures, the Fubini theorem.
12. Absolute continuity of measures and real functions, singular measures, the Lebesgue-Radon-Nikodým theorem, properties of Radon-Nikodým derivatives.
13. Multi-dimensional distribution functions, multi-dimensional Lebesgue-Stieltjes measures and integrals, infinite dimensional product probability measure, Kolmogorov consistency theorem.
14. Sigma algebras generated by random variables, independence, Borel-Cantelli theorem and Borel zero-one law, tail events, Kolmogorov zero-one law.

OUTSIDE READINGS: There will not be any required outside readings. However, the student is definitely encouraged to refer, from time to time, to the following books and/or papers. Many of these are regarded as being classics in the field. Of the textbooks in measure, integration, and probability theory listed, Chow and Teicher [6] probably has a development best matching the one in this course. A copy of this text in in the John Saw Statistics Library, but it is not to be checked out. (This list also pertains to STA 6467.)

1. Billingsley, P. (1968). Convergence of Probability Measures. Wiley, New York.
2. Billingsley, P. (1995). Probability and Measures (3rd ed.). Wiley, New York.

3. Breiman, L. (1968). Probability. Addison-Wesley, Reading, Mass.

4. Chow, Y.S., and Robbins, H. (1961). On sums of independent random variables with infinite moments and "fair" games. Proc. Nat. Acad. Sci. USA. 47, 330-335

5. Chow, Y.S., Robbins, H., and Siegmund, D. (1971). Great Expectations: The Theory of Optimal Stopping. Houghton Mifflin, Boston.

6. Chow, Y.S. and Teicher, H. (1997). Probability Theory: Independence, Interchangeability, Martingales (3rd ed.). Springer-Verlag, New York.

7. Chung, K.L. (1974). A Course in Probability Theory (2nd ed). Academic Press, New York.

8. Doob, J.L. (1953). Stochastic Processes. Wiley, New York.

9. Feller, W. (1945). The fundamental limit theorems in probability. Bull. Amer. Math. Soc. 51, 810-832.

10. Feller, W. (1946). A limit theorem for random variables with infinite moments. Amer. J. Math. 68. 257-262.

11. Feller, W. (1971). An Introduction to Probability Theory and Its Applications. Vol. II (2nd ed.). Wiley, New York.

12. Galambos, J. (1988). Advanced Probability Theory. Marcel Dekker, New York.

13. Gnedenko, B.V. and Kolmogorov, A.N. (1968). Limit Distributions for Sums of Independent Random Variables (Translated from the Russian, annotated, and revised by K.L. Chung with appendices by J.L. Doob and P.L. Hsu). Addison-Wesley, Reading, Mass.

14. Halmos, P.R. Measure Theory. Van Nostrand, Princeton, 1950; Springer-Verlag, Berlin and New York, 1974.

15. Hewitt, H. and Stromberg, K. (1975). Real and Abstract Analysis-A Modern Treatment of the Theory of Functions of a Real Variables. Springer-Verlag, Berlin and New York.

16. Ibragimov, I.A. and Linnik, Yu, V. (1971). Independent and Stationary Sequences of Random Variables (Translated from the Russian). Wolters-Noordhoff, Groningen.

17. Kesten, H. (1972). Sums of independent random variables-without moment conditions. Ann. Math. Statist. 43, 701-732.

18. Kingman, J.F.C. and Taylor, S.J. (1966). Introduction to Measure and Probability. Cambridge University Press, Cambridge.

19. Kolmogorov, A.N. (1933). Foundations of the Theory of Probability (1956 Second English Edition translated from the German and edited by N. Morrison with an added bibliography by A.T. Bharucha-Reid). Chelsea, New York.

20. Krickeberg, K. (1965). Probability Theory. Addison-Wesley, Reading Mass.

21. Laha, R.G. and Rohatgi, V.K. (1979). Probability Theory. Wiley, New York.

22. Lévy, P. (1953). Loi faible et loi forte des grands nombres. Bull. des Sci. Math. 77, 9-40.

23. Lévy, P. (1954). Théorie de l'Addition des Variables Aléatoires (2nd ed.). Gauthier-Villars, Paris.

24. Loève, M. (1963). Probability Theory (3rd ed.). Van Nostrand, Princeton; (1977-1978) Probability Theory, Vol. I and II (4th ed.). Springer-Verlag, Berlin and New York.

25. Lukacs, E. (1970). Characteristic Functions (2nd ed.). Hafner, New York.

26. Marcinkiewicz, J. and Zygmund, A. (1937). Sur les fonctions indépendantes. Fund. Math. 29, 60-90.

27. Neveu, J. (1965). Mathematical Foundations of the Calculus of Probability (Translated from the French). Holden-Day, San Francisco.

28. Petrov, V.V. (1975). Sums of Independent Random Variables (Translated from the Russian). Springer-Verlag, Berlin and New York.

29. Petrov, V.V. (1995). Limit Theorems of Probability Theory: Sequences of Independent Random Variables. Clarendon Press, Oxford.

30. Rao, M.M. (1984). Probability Theory with Applications. Academic Press, Orlando.

31. Rao, M.M. (1987). Measure Theory and Integration. Wiley, New York.

32. Rényi, A. (1970). Probability Theory. (Translated from the Hungarian). North-Holland, Amsterdam and London.

33. Rényi, A. (1970). Foundations of Probability. Holden-Day, San Francisco.

34. Révesz, P. (1968). The Laws of Large Numbers. Academic Press, New York.

35. Royden, H.L. (1968). Real Analysis (2nd ed.). Macmillan, New York.

36. Rudin, W. (1966). Real and Complex Analysis. McGraw-Hill, New York.

37. Saks, S. (1937). Theory of the Intgral. Stechert-Hafner, New York.

38. Seneta, E. (1976). Regularly Varying Functions. Springer-Verlag, Berlin and New York.

39. Shiryayev, A.N. (1984). Probability (Translated from the Russian by R.P. Boas). Springer-Verlag, Berlin and New York.

40. Spitzer, F. (1976). Principles of Random Walk (2nd ed.). Springer-Verlag, Berlin and New York.

41. Stout, W.F. (1974). Almost Sure Convergence. Academic Press, New York.

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CLASSROOM ACCOMMODATIONS: Students requesting classroom accommodations must first register with the Dean of Students Office. The Dean of Students Office will provide the student a letter to be given to the instructor when requesting accommodations.