Course Syllabus

STA 7467 Probability Theory 2 (Section 7017) Spring 2011

CLASS: MWF Period 7 (1:55p.m. - 2:45 p.m.) FLO 230

INSTRUCTOR:

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OFFICE HOURS: Period 6 (MWF) and Period 9 (MW)

COURSE OBJECTIVE: The two-semester sequence STA 7466–7467 covers that material from measure, integration, and probability theory that every statistics doctoral student should know. Since the axiomatic approach to probability theory (which was developed by the eminent Russian probabilist Andrei N. Kolmogorov and published in Germany in 1933) rests on the theory of abstract measure and integration, the first semester is devoted mainly to this theory. During the second semester, we will apply our knowledge of measure and integration theory for studying independence; modes of convergence of random variables; convergence of series of independent random variables; strong and weak laws of large numbers; weak and complete convergence of distribution functions; characteristic functions including the uniqueness, inversion, and continuity theorems; the central limit problem.

COURSE REQUIREMENTS:(Six Exams)
Exam 1 (in class: 50 points)
Exam 2 (take-home: 50 points)
Exam 3 (in class: 50 points)
Exam 4 (take-home: 50 points)
Exam 5 (in class on Wednesday April 20, 2011: 50 points)
Exam 6 (take-home, due Friday April 22, 2011 by 3:00pm (Bring it to my office between 1:00pm and 3:00pm.): 50 points)

Exams 1 and 2 will be more or less at the same times; so will be Exams 3 and 4 and also Exams 5 and 6. There will also be several homework assigements. Exam questions will consist of original problems (proofs required) as well as questions calling for the construction of examples or counterexamples elucidating the fine points or subtleties of the theory (proofs required).

REQUIRED COURSE MATERIAL: "Notes on Measure, Integration, and Probability Theory" (revised Summer 2010) by Andrew Rosalsky.

LIST OF TOPICS:

1. Modes of convergence of random variables and their relationship, Lévy's inequality, Lévy's theorem on the equivalence between convergence in probability and almost certain convergence for series of independent random variables.

2. Summability of independent random variables, Khintchine-Kolmogorov convergence theorem, Abel summation by parts, the Kronecker lemma, tail (or Khintchine) equivalence, Kolmogorov three-series criterion, Marcinkiewicz-Zygmund convergence theorem.

3. Loève Strong Law of Large Numbers (SLLN), Marcinkiewicz-Zygmund SLLN, Kolmogorov SLLN, application of the strong law to Monte Carlo simulation, the asymptotic fluctuation behavior of $X_n/n^{1/p}$, the asymptotic fluctuation behavior of X_n/a_n and S_n/a_n, statistical point estimation and strong consistency and the Komlös-Révész example.

4. Chebyshev Weak Law of Large Numbers (WLLN), Bernstein's proof of the Weierstrass approximation theorem via the weak law, Feller WLLN, Khintchine WLLN, Kolmogorov inequality, the weak law for random indices.

5. Method of subsequences, weak symmetrization inequalities, Lemma for Events, strong symmetrization inequalities, Toeplitz lemma, Prokhorov-Loève almost certain stability criterion.

6. Weak convergence, complete convergence, convergence in distribution, Slutsky's theorem, Skorokhod representation theorem, the finite and extended Helly-Bray theorems, uniform integrability, Scheffé's theorem on convergence of densities.

7. Helly's weak compactness theorem, complete compactness, Fréchet-Shohat theorem, the moment problem, empirical distribution functions and the Glivenko-Cantelli theorem, Khintchine convergence of types theorem, convolution.

8. Characteristic functions (Fourier-Stieltjes transforms), Lévy inversion formula, Lévy uniqueness theorem, Fourier inversion theorem, correspondence between convolution of distribution functions and multiplication of characteristic functions, Lévy continuity theorem, the equivalence between convergence in distribution and convergence almost certainly for series of independent random variables.

9. Derivatives of characteristic functions and moments, Polya's criterion, a necessary criterion for convergence in distribution of normalized sums of random variables.

10. Central limit problem, Lindeberg condition, Lindeberg-Feller classical normal convergence criterion (Central Limit Theorem (CLT)), Lévy CLT, Liapounov CLT, Berry-Esseen theorem, the asymptotic fluctuation behavior of $S_n/\sqrt{n}$.

11. Additional topics if time is available.

<u>**ACADEMIC HONESTY**</u>: University of Florida students are expected to abide by the following: "We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity."

<u>**CLASSROOM ACCOMMODATIONS**</u>: Students requesting classroom accommodations must first register with the Dean of Students Office. The Dean of Students Office will provide the student a letter to be given to the instructor when requesting accommodation.