

Prove that a point of inflection is one standard deviation away from the center (mean) of a normal distribution.

$$y = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$y' = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \cdot \frac{-2(x-\mu)}{2\sigma^2}$$

$$= \frac{-1}{\sigma^2 \sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} (x-\mu)$$

$$y'' = \frac{-1}{\sigma^2 \sqrt{2\pi\sigma^2}} \left[e^{-\frac{(x-\mu)^2}{2\sigma^2}} \left(\frac{-2(x-\mu)}{2\sigma^2} \right) (x-\mu) + e^{-\frac{(x-\mu)^2}{2\sigma^2}} \right]$$

For a pt of inflection $y'' = 0$

$$0 = \frac{-1}{\sigma^2 \sqrt{2\pi\sigma^2}} \left[\frac{-e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sigma^2} (x-\mu)^2 + e^{-\frac{(x-\mu)^2}{2\sigma^2}} \right]$$

$$\frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sigma^2} (x-\mu)^2 = e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$(x-\mu)^2 = \sigma^2$$

$$x-\mu = \pm \sigma$$

$$x = \mu \pm \sigma$$

Q.E.D