

STA 3032
Section 7370
Quiz #4
Spring, 2009

1. Horizontal (x) and vertical (y) expansion were measured on 9 bridges in the area of Quebec City:

Bridge	1	2	3	4	5	6	7	8	9
x	20	15	43	5	18	24	32	10	21
y	68	58	55	75	58	68	55	69	63

Summary statistics are:

$$\bar{x} = 20.9 \qquad \bar{y} = 60.9$$

$$S_{xx} = \sum (x - \bar{x})^2 = 1036.9 \qquad S_{yy} = \sum (y - \bar{y})^2 = 524.9$$

$$S_{xy} = \sum (x - \bar{x})(y - \bar{y}) = -515.1$$

a. (5) The equation for the least squares line is $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x = 71.4 - 0.50x$. Use the **summary statistics** to illustrate computation of $\hat{\beta}_0$ and $\hat{\beta}_1$:

$$\hat{\beta}_1 = S_{xy} / S_{xx} = -515.1 / 1036.9 = -0.497 \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1(\bar{x}) = 60.0 + 0.497(20.9) = 70.382$$

b. (5) Complete the following analysis of variance (ANOVA) table:

Source	df	SS	MS
Regression	1	255.9	255.9
Error	7	269.0	38.43
Total	8	524.9	

c. (3) $R^2 = .49$. Use results in the ANOVA table to show how R^2 was computed:

$$R^2 = .49 = \mathbf{255.9 / 524.9}$$

d. (5) Construct a 95% confidence interval for β_1 :

$$\hat{\beta}_1 \pm t_{.025,7} \sqrt{MSE / S_{xx}} = -0.497 \pm 2.36 \sqrt{38.43 / 1036.9} = -0.497 \pm 0.454$$

e. (2) Compute the correlation coefficient between horizontal and vertical expansion:

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{-515.1}{\sqrt{(1036.9)(524.9)}} \quad \text{Also, } r = \text{negative square root of } R^2, \text{ } -.70$$

f. (3) The method least squares is used to **obtain parameter estimates**, $\hat{\beta}_0$ and $\hat{\beta}_1$.

g. (2) The least squares estimates are obtained by minimizing

$$\sum_i (y_i - (b_0 + b_1 x_i))^2 \text{ with respect to } b_0 \text{ and } b_1.$$