

## Practice Problems – Random Coefficient Regression

Q.1. Random Coefficient Regression:

$$Y_{ij} = \beta_{0i} + \beta_{1i}X_{ij} + \varepsilon_{ij} \quad \begin{bmatrix} \beta_{0i} \\ \beta_{1i} \end{bmatrix} \sim NID\left(\begin{bmatrix} \beta_{0i} \\ \beta_{1i} \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{01} & \sigma_1^2 \end{bmatrix}\right) \quad \varepsilon_{ij} \sim NID(0, \sigma^2) \quad \left\{ \begin{bmatrix} \beta_{0i} \\ \beta_{1i} \end{bmatrix} \right\} \perp \{ \varepsilon_{ij} \}$$

Derive  $V\{Y_{ij}\}$ .

Q.2. For the following model, give the variance-covariance matrix of  $\mathbf{Y}$ :

$$Y_{ij} = \alpha_i + \beta_i X_{ij} + \varepsilon_{ij} \quad i = 1, 2 \quad j = 1, 2, 3 \quad \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \sim \left[ \begin{array}{c} \left( \begin{array}{c} \alpha \\ \beta \end{array} \right), \left( \begin{array}{cc} \sigma_\alpha^2 & \sigma_{\alpha\beta} \\ \sigma_{\alpha\beta} & \sigma_\beta^2 \end{array} \right) \end{array} \right]$$

Q.3. A growth curve model is being fit for  $n$  subjects, with  $t$  measurements per subject. The following model is being fit.

$$Y_{ij} = \alpha_i + \beta_i X_{ij} + \varepsilon_{ij} \quad i = 1, \dots, n; j = 1, \dots, t \quad \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \sim NID\left(\begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \begin{bmatrix} \sigma_\alpha^2 & \sigma_{\alpha\beta} \\ \sigma_{\alpha\beta} & \sigma_\beta^2 \end{bmatrix}\right) \quad \varepsilon_{ij} \sim NID(0, \sigma^2) \quad \left\{ \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \right\} \perp \{ \varepsilon_{ij} \}$$

Give  $E\{Y_{ij}\}$ ,  $V\{Y_{ij}\}$ ,  $\text{COV}\{Y_{ij}, Y_{i'j'}\} \quad j \neq j'$