STA 6208 – Homework 2

Part 1: Problems

Q.1. An experiment is conducted to compare 3 equally spaced dryer temperatures on fabric shrinkage. The researcher samples 15 pieces of wool fabric (labeled specimen1-specimen15). He generates random numbers for each specimen, then assigns 5 to each treatment (the 5 specimens with the smallest random numbers are assigned to temperature 1, the 5 specimens with the largest random numbers receive temperature 3, and others receiving temperature 2).

spec1	spec2	spec3	spec4	spec5	spec6	spec7	spec8	spec9	spec10	spec11	spec12	spec13	spec14	spec15
0.541239	0.849694	0.460164	0.608456	0.202543	0.331311	0.186567	0.416428	0.442315	0.278932	0.699956	0.67784	0.197721	0.662758	0.799943

p.1.a. Specimens receiving Temp1 ________________ Temp2 ________________ Temp3 ____________________

p.1.b. The means and standard deviations for the amount of shrinkage for the three temperatures are given below. Compute the Treatment and Error Sums of Squares:

Temp	r	Mean	SD
1	5	6	3
2	5	14	2
3	5	16	3

SS_{TRT} =

SS_{ERR} =

p.1.c. Complete the following ANOVA table.

Source	df	Sum of Squares	Mean Square	F_0	F(.05)
Treatments					
Error					
Total					

p.1.d. Consider 2 Contrasts: $C_{Lin} = (-1)\mu_1 + (0)\mu_2 + (1)\mu_3$ and $C_{Quad} = (1)\mu_1 + (-2)\mu_2 + (1)\mu_3$

p.1.d.i. Show that these contrasts are orthogonal.

p.1.d.ii. Give the sums of squares for these contrasts and show they sum to SS_{TRT}

$SS(C_{Lin})$ =

$SS(C_{Quad})$ =

Q.2. An experimenter has g=8 methods of preparing steel rods from raw steel, and is interested in comparing their mean breaking strengths. She obtains 40 batches of steel, and randomly assigns them, so that batches are used for each method (that is, n=5). Before conducting the experiment, she envisions many potential comparisons (contrasts) among the treatments and decides she will use Scheffe's method to conduct all her tests concerning the contrasts (with experimentwise error rate of $\alpha_E = 0.05$). Suppose here Error Sum of Squares is SSE = 200. How large will a Contrast sum of squares need to be to conclude that the contrast among population means is not equal to 0 (reject the null hypothesis that the contrast is 0)?

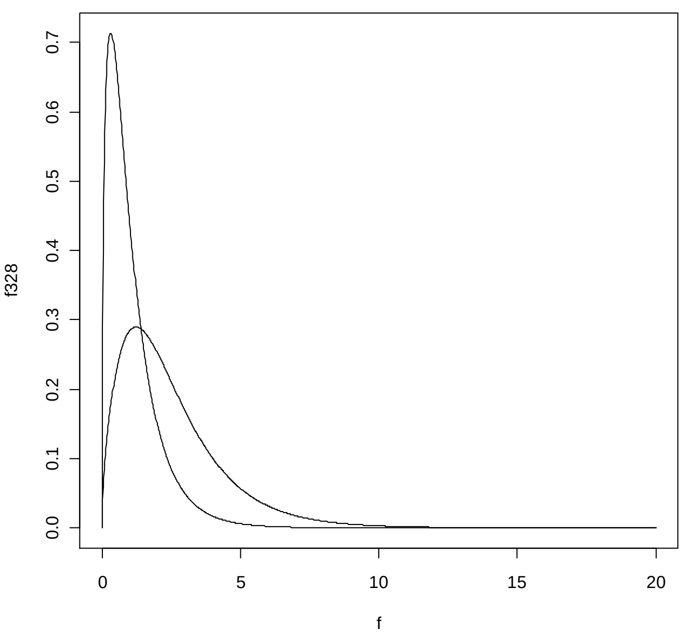
Q.3.. Compute Tukey's and Bonferroni's minimum significant differences (with experimentwise error rates of $\alpha_E = 0.05$) when the experiment consists of 5 treatments with, with 4 replicates per treatment and SSE = 400.

Q.4. We wish to conduct an experiment to compare t=4 treatments in a CRD. We would like the probability that we (correctly) reject the null hypothesis to be $1-\beta = 0.80$ when the test is conducted at $\alpha = 0.05$ and the α_i are -20,0,0,20 and $\sigma = 40$.

p.4.a. What is the non-centrality parameter in this setting when n=4, when n=8?

p.4.b. What is the Rejection Region for the test when n=4, when n=8?

p.4.c. Identify on the following plot, the rejection region and power of the F-test for the case n=8. The distribution to the "left" is the central F, the distribution to the "right" is the non-central F.



p.4.d. Does it appear that the power has reached .80 for n=8?

Q.5. Two statistical programs are fitting a 1-Way Analysis of Variance, based on the treatment effects model:

$$y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \quad i=1,..,4 \quad j=1,...,n \quad \text{Program A: } \alpha_4 = 0 \quad \text{Program B: } \sum_{i=1}^{4} \alpha_i = 0$$

The least squares estimates of the model parameters: μ, τ_1, τ_2, τ_3 are given below:

Parameter	Program A	Program B
μ	60	55
$\alpha 1$	-10	-5
$\alpha 2$	10	15
$\alpha 3$	-20	-15

p.5.a. Compute the least squares estimates of the following estimable parameters, based on each program (Show all numbers used in calculations):

p.5.a.i.: $\mu + \alpha_1$ Program A ______________________________ Program B ________________________________

p.5.a.ii.: $\mu + \alpha_4$ Program A ______________________________ Program B ________________________________

p.5.a.iii.: $\alpha_2 - \alpha_1$ Program A ______________________________ Program B ________________________________

p.5.a.iv.: $\alpha_4 - \alpha_1$ Program A ______________________________ Program B ________________________________

p.5.b. For this experiment, what is the largest the error sum of squares could be, for us to reject H_0: $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$, if n = 5?

Q.6. For a one-way fixed effects analysis with t=3 treatments and unequal sample sizes (n_1, n_2, n_3), derive the ordinary least squares estimators for the treatment effects model. Show all work.

$$y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \quad i=1,2,3 \quad j=1,...,n_i \quad \sum_{i=1}^{3} \alpha_i = 0$$

p.6.a. Derive the Ordinary Least Squares Estimator of μ:

p.6.b. Derive the Ordinary Least Squares Estimator of α_1:

p.6.c. Obtain $E\left\{Y_{ij}^2\right\}, \quad E\left\{\overline{Y}_{i\bullet}^2\right\}, \quad E\left\{\overline{Y}_{\bullet\bullet}^2\right\}$

Q.7. A 1-Way ANOVA is to be fit with g = 3 treatments and sample sizes $n_1 = 2$, $n_2 = 4$, $n_3 = 3$

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} = \mu_i + \varepsilon_{ij} \quad i = 1,2,3;\ j = 1,\ldots,n_i \qquad \underset{N\times 1}{\mathbf{Y}} = \underset{N\times 3}{\mathbf{X}}\,\underset{3\times 1}{\boldsymbol{\beta}} + \underset{N\times 1}{\boldsymbol{\varepsilon}}$$

Give the form of the **X** matrix, **X'X** matrix and $\boldsymbol{\beta}$ vector for each of the following parameterizations.

p.7.a. $\mu^* = 0 \quad \alpha_i^* = \mu_i \quad i = 1,2,3$

p.7.b. $\alpha_1^* = 0 \quad \mu^* = \mu_1 \quad \alpha_i^* = \mu_i - \mu_1 \quad i = 1,2,3$

p.7.c. $\mu^* = \mu \quad \sum_{i=1}^{3} \alpha_i^* = 0$

Q.8. A published report, based on a balanced 1-Way ANOVA reports means (SDs) for the three treatments as:

Trt 1: 70 (8) Trt 2: 75 (6) Trt 3: 80 (10)

Unfortunately, the authors fail to give the sample sizes.

p.8.a. Complete the following table, given arbitrary levels of the number of replicates per treatment:

r	SSTrt	SSErr	MSTrt	MSErr	F_obs	F(.05)
2						
6						
10						

p.8.b. The smallest n, so that these means are significantly different is:

i) n <= 2 ii) 2 < n <= 6 iii) 6 < n <= 10 iv) n > 10

Q.9. For the balanced completely randomized design with g treatments, and n units per treatment, consider the following (treatment effects) model:

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \quad i = 1,\ldots,g;\ j = 1,\ldots,n \quad \sum_{i=1}^{g} \alpha_i = 0 \quad \varepsilon_{ij} \sim NID\left(0,\sigma^2\right)$$

p.9.a. Derive the least squares estimate of μ: $\hat{\mu}$ SHOW ALL WORK

p.9.b. Derive the least squares estimate of α_k: $\hat{\alpha}_k$ SHOW ALL WORK

p.9.c. Derive: $E\left\{\hat{\mu}\right\},\ V\left\{\hat{\mu}\right\},\ E\left\{\hat{\alpha}_k\right\},\ V\left\{\hat{\alpha}_k\right\},\ COV\left\{\hat{\mu},\hat{\alpha}_k\right\},\ COV\left\{\hat{\alpha}_k,\hat{\alpha}_{k'}\right\}\ \left(k \neq k'\right)$ SHOW ALL WORK

Part 2: Comparison of Three Methods of Teaching Drawing to Children – Summary Stats

Experiment to compare effects of 3 methods on improving drawing:

- Edwards' Training Procedure (ET), (n=19, Mean=7.02, SD=3.26)
- placebo control group involving a sham (nonsensical) treatment (ST), (n=18, Mean=7.90, SD=3.03)
- waiting list control group (WC). (n=16, Mean=2.40, SD=3.12)

Scores are differences in drawing rating scores (post tx - pre tx).

- Conduct a 1-Way ANOVA and obtain 95% CI's for population means of each of the methods.
- Use the following two orthogonal contrasts that partition the Treatment sum of squares into: ET vs ST, and (ET,ST vs WC). Show that these sum of squares sum to SSTreatments. Note: due to unequal sample sizes, you can use the following Contrast vectors (show they are orthogonal). Obtain 95% Confidence Intervals, and conduct the F-test for each contrast.

$C_1' = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} \qquad C_2' = \begin{bmatrix} 19 & 18 & -37 \end{bmatrix}$

- Use Tukey's, Bonferroni's and Scheffe's methods to compare all pairs of treatments with experimentwise error rates of α_E=0.05.

Part 3: Simulate 10000 random samples from each of the following models, and obtain the approximate probability of rejecting H_0: $\mu_1 = \mu_2 = \mu_3$ at α = 0.05 significance level.

$$y_{ij} = \mu_i + e_{ij} \quad e_{ij} \sim NID\left(0, \sigma_i^2\right)$$

$\mu 1$	$\mu 2$	$\mu 3$	$\sigma 1$	$\sigma 2$	$\sigma 3$	n1	n2	n3
100	100	100	10	10	10	10	10	10
100	100	100	20	10	5	15	10	5
100	100	100	20	10	5	5	10	15
90	100	110	10	10	10	10	10	10
90	100	110	20	10	5	15	10	5
90	100	110	20	10	5	5	10	15