Comparing 2 Treatments

Q.1. An accounting researcher is interested in comparing two methods of training auditors for preparing tax returns. She wants to choose equal sample sizes for a 2-sample t-test has a power of 0.90 of detecting a difference in true mean tax assessment of 5. Based on a pilot study, she believes the standard deviation is about 15. Note that auditors will be using the same corporate financial data. Complete the following parts, and show all work in parts b-d.

Note: $z_{.10} = 1.28$ $z_{.05} = 1.645$ $z_{.025} = 1.96$

p.1.a. Assuming $Y_{ij} \sim NID\left(\mu_i, \sigma^2\right)$ give the sampling distribution of $\overline{Y}_{1\bullet} - \overline{Y}_{2\bullet}$

p.1.b. Based on the normal distribution, for what values of $\overline{Y}_{1\bullet} - \overline{Y}_{2\bullet}$ will you reject H_0: $\mu_1 - \mu_2 = 0$ in favor of H_A: $\mu_1 - \mu_2 \neq 0$ based on a Z-test? Note given the above information, this will be a function of $n_1 = n_2 = n$.

.1.c. What will the (upper) critical value from part b be equal to under the alternative hypothesis $\mu_1 - \mu_2 = 5$, if you want the power to be 0.90 (equivalently β = P(Type II Error) = 0.90). Note given the above information, this will be a function of $n_1 = n_2 = n$.

p.1.d. What sample size ($n_1 = n_2 = n$) will be needed for each group to equate parts b and c?

Q.2. An experiment to compare 2 treatment means is conducted as a paired experiment. The summary data are:

$$n = 16 \quad \overline{y}_1 = 52 \quad s_1 = 12 \quad \overline{y}_2 = 45 \quad s_2 = 15$$

Technically, this data could be analyzed as an independent sample t-test (ignoring the pairing) or a paired t-test. How large would the sample covariance between the measurements within pairs need to be for the 95% Confidence Interval for $\mu_1 - \mu_2$ to be narrower based on the paired sample approach than the independent samples approach?

Q.3. An investigator wishes to compare the variances of the purity of 2 brands of a chemical product. The experiment will consist of obtaining independent samples of $n_1 = n_2 = 7$ batches from each brand. How large would the ratio of the larger sample standard deviation to the smaller sample standard deviation need to be to reject $H_0 : \sigma_1^2 = \sigma_2^2$ in favor of $H_A : \sigma_1^2 \neq \sigma_2^2$ at the $\alpha = 0.10$ significance level.

Q.4. A textile engineer is interested whether the mean breaking strength of Yarn Type A is larger than the of Type B. Assuming that the sample standard deviations are 10, how large should each sample size be if we want P(Reject H_0: $\mu_A - \mu_B = 0$ in favor of H_A: $\mu_A - \mu_B > 0 \mid \mu_A - \mu_B = 2) = 0.95$. **Show all work based on the normal distribution**.