STA 6208 – Exam 3 – Spring 2017 **PRINT** **NAME ______________**

Conduct all tests at $\alpha = 0.05$ significance level

For the following scenarios, give the Sources of Variation, degrees of freedom, and appropriate error term for its corresponding F-test.

S.1. An experiment involves a taste comparison among the only 4 varieties of red wines produced by a family vineyard. A sample of 10 wine tasters from the city's wine tasting association is obtained. Each rater tastes each variety 5 times (blind, and in random order).

Source	df	Error Mean Square Term

S.2. A textile company has 5 manufacturing plants. Within each plant there are many machine operators, and they sample 4 from each plant. There are 3 models of sewing machines used at the plants (the models are used at all plants, and each operator is trained for each machine). Each operator in the study makes 2 production runs on each machine, and the quality of each product is measured.

Source	df	Error Mean Square Term

Q.1. An experiment is to be conducted as a Randomized Complete Block Design with *t* fixed treatments, in *b* random blocks.

$$Y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \quad i = 1,...,t;\ j = 1,...,b \quad \sum_{i=1}^{t} \tau_i = 0 \quad \beta_j \sim NID\left(0, \sigma_\beta^2\right) \quad \varepsilon_{ij} \sim NID\left(0, \sigma^2\right) \quad \left\{\beta_j\right\} \perp \left\{\varepsilon_{ij}\right\}$$

p.1.a. Give the Covariance structure for the measurements $\{Y_{ij}\}$.

p.1.b. Derive $V\left\{\overline{Y}_{i\bullet}\right\}$, $\mathrm{COV}\left\{\overline{Y}_{i\bullet}, \overline{Y}_{i'\bullet}\right\}$ $\left(i \neq i'\right)$, $V\left\{\overline{Y}_{i\bullet} - \overline{Y}_{i'\bullet}\right\}$ Show all work.

Q.2. An experiment is conducted as a Latin Square with $t = 4$ treatments. The row factor is random and the column factor is fixed. If squares are replicated, with different levels of the row factor in each square, how many squares will be needed to assure there are at least 50 degrees of freedom for error. Show calculations for each number of squares, 1,2,…

Q.3. Mixed model with factor A fixed with $a = 2$ levels and factor B random with 3 levels, random interaction, and $n = 2$ replicates per combination. Give the model in terms of $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{u} + \boldsymbol{\varepsilon}$. Give all elements in full form.

Q.4. A Balanced Incomplete Block Design is conducted, to compare 7 (fixed) machines in terms of output. The supplier provides (random) batches (blocks) of components that only have 4 components.

$Y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \quad i = 1,...,t; \ j = 1,...,b$ Note: not all pairs (i, j)

p.4.a. How many batches will be needed so that each machine produces 4 items? Fill in the following numbers, where t = #trts, k = block size, r = # reps/trt, b = # blocks, λ = # blocks each pair of treatments appears in together.

$t =$ ____________ $k =$ ____________ $r =$ ____________ $b =$ ____________ $\lambda =$ ____________

p.4.b. The error sum of squares for a model that contains only blocks as a factor is 200. The error sum of squares for a model that contains both blocks and treatments as factors is 120. Test for treatment effects (adjusted for blocks). $H_0: \tau_1 = ... = \tau_\tau = 0$.

Test Statistic: ____________________________ Rejection Region: _____________________________

p.4.c. We wish to obtain simultaneous 95% Confidence Intervals for all pairs of treatment differences based on all the intra-block analysis, where: $\hat{V}\left\{\hat{\tau}_i - \hat{\tau}_{i'}\right\} = \frac{2kMS_{\text{ERR}}}{\lambda t}$. Give the form of the simultaneous 95% CI's:

$$\left(\hat{\tau}_i - \hat{\tau}_{i'}\right) \pm$$

Q.5. A repeated measures design is conducted to compare 3 treatments over 3 time points, with 10 subjects per treatment. The model is:

$$Y_{ij} = \mu + \alpha_i + \beta_{j(i)} + \tau_k + (\alpha\tau)_{ik} + \varepsilon_{jk(i)} \quad i = 1,2,3,\ j = 1,\ldots,10; k = 1,2,3$$

$$\beta_{j(i)} \sim NID\left(0,\sigma_\beta^2\right) \quad \varepsilon_{jk(i)} \sim NID\left(0,\sigma^2\right) \quad \{\beta\} \perp \{\varepsilon\} \quad \sum_i \alpha_i = \sum_k \tau_k = \sum_i (\alpha\tau)_{ik} = \sum_k (\alpha\tau)_{ik} = 0$$

p.5.a. Under this model, give the within subject variance-covariance matrix.

p.5.b. The treatment means are: $\overline{y}_{1\bullet\bullet} = 90.6 \quad \overline{y}_{2\bullet\bullet} = 95.3 \quad \overline{y}_{3\bullet\bullet} = 108.7$. Complete the following ANOVA table.

Source	df	SS	MS	F	F(.95)
Trts					
Subj(Trt)		11360		#N/A	#N/A
Time		727			
TrtxTime		279			
Error=Time*S(Trt)				#N/A	#N/A
Total		18473		#N/A	#N/A

p.5.c. Assuming there is a significant interaction, based on Bonferroni's method, test for significant differences among all pairs of treatments at the 3rd time point

$$\overline{Y}_{1\bullet 3} = 102.6 \quad \overline{Y}_{2\bullet 3} = 108.1 \quad \overline{Y}_{3\bullet 3} = 102.6 \quad \hat{V}\left\{\overline{Y}_{i\bullet k} - \overline{Y}_{i'\bullet k}\right\} = \frac{2\left(MS_{B(A)} + (t-1)MS_{\mathrm{ERR2}}\right)}{nt}$$

p.5.d. Give the ANOVA estimates for σ^2 and σ_β^2

Q.6. A study is conducted to compare pH levels in rivers in the 3 geographic areas of a state. Random samples of 5 rivers were selected within each of the geographic areas, and 4 replicates were obtained within each river.

p.6.a. Write out the statistical model. Be very specific.

p.6.b. Complete the following Analysis of Variance table.

Source	df	SS	MS	F	F(.95)
Area		4000			
River w/in Area		2400			
Error		2250			
Total					

p.6.c. Compute Bonferroni's minimum significant difference for comparing all pairs of geographic areas.