STA 6208 – Exam 1 – Spring 2017 PRINT NAME ______________

Conduct all tests at the α = 0.05 significance level unless stated otherwise.

Q.1. An experiment to compare 2 treatment means is conducted as a paired experiment. The summary data are:

$$n = 16 \quad \bar{y}_1 = 52 \quad s_1 = 12 \quad \bar{y}_2 = 45 \quad s_2 = 15$$

Technically, this data could be analyzed as an independent sample t-test (ignoring the pairing) or a paired t-test. How large would the sample covariance between the measurements within pairs need to be for the 95% Confidence Interval for $\mu_1 - \mu_2$ to be narrower based on the paired sample approach than the independent samples approach?

Q.2. An investigator wishes to compare the variances of the purity of 2 brands of a chemical product. The experiment will consist of obtaining independent samples of $n_1 = n_2 = 7$ batches from each brand. How large would the ratio of the larger sample standard deviation to the smaller sample standard deviation need to be to reject $H_0: \sigma_1^2 = \sigma_2^2$ in favor of $H_A: \sigma_1^2 \neq \sigma_2^2$ at the $\alpha = 0.10$ significance level.

Q.3. For the balanced 1-Way Analysis of Variance model, complete the following parts.

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \quad i = 1, \ldots, g;\ j = 1, \ldots, n \quad \varepsilon_{ij} \sim NID\left(0, \sigma^2\right)$$

p.3.a. Show that: $SS_{\text{Err}} = \sum_{i=1}^{g} \sum_{j=1}^{n} Y_{ij}^2 - n \sum_{i=1}^{g} \overline{Y}_{i\bullet}^2$ and $SS_{\text{Trts}} = n \sum_{i=1}^{g} \overline{Y}_{i\bullet}^2 - N \overline{Y}_{\bullet\bullet}^2$

p.3.b. Derive $E\left\{SS_{\text{Err}}\right\}$ and $E\left\{SS_{\text{Trts}}\right\}$

Q.4. A study compared the antioxidant activity for 4 varieties of green tea. Five replicates were obtained from each tea variety and the total phenolic content was measured. The means and standard deviations are given below.

Variety	1	2	3	4
Mean	160	170	140	190
SD	15	18	12	15

p.4.a. Complete the following Analysis of Variance table.

Source	df	Sum of Squares	Mean Square	F_obs	F(.95)
Treatments					
Error					
Total					

p.4.b. Use Tukey's Method to compare all pairs of variety means.

p.4.c. Compute Bonferroni's and Scheffe's minimum significant differences for comparing all pairs of treatments.

Q.5. A 1-Way ANOVA is to be fit with g = 3 treatments and sample sizes $n_1 = 2$, $n_2 = 4$, $n_3 = 3$

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} = \mu_i + \varepsilon_{ij} \quad i = 1,2,3;\ j = 1,\ldots,n_i \qquad \underset{N\times 1}{\mathbf{Y}} = \underset{N\times 3}{\mathbf{X}}\ \underset{3\times 1}{\boldsymbol{\beta}} + \underset{N\times 1}{\boldsymbol{\varepsilon}}$$

Give the form of the **X** matrix, **X'X** matrix and $\boldsymbol{\beta}$ vector for each of the following parameterizations.

p.5.a. $\mu^* = 0 \quad \alpha_i^* = \mu_i \quad i = 1,2,3$

p.5.b. $\alpha_1^* = 0 \quad \mu^* = \mu_1 \quad \alpha_i^* = \mu_i - \mu_1 \quad i = 1,2,3$

p.5.c. $\mu^* = \mu \quad \sum_{i=1}^{3} \alpha_i^* = 0$

Q.6. A study was conducted to compare the effects of 4 evenly spaced doses of a drug on a measured response. There were 6 replicates per dose, sample mean responses were 10, 18, 22, and 26 respectively and $SS_{Err} = 500$.

p.6.a. Compute the Treatment sum of Squares, SS_{Trts}.

p.6.b. The goal was to partition the Treatment sum of squares into Linear, Quadratic, and Cubic components.

i	Linear	Quadratic	Cubic
1	-3	1	-1
2	-1	-1	3
3	1	-1	-3
4	3	1	1

Making use of the pairwise orthogonal contrasts, compute SS_{Lin}, SS_{Quad}, SS_{Cubic} and show they sum to SS_{Trts}

p.6.c. Compute the F-statistics and the critical F value (do not adjust for multiple tests) for testing these 3 (population) contrasts are 0.

F_{Lin} = ________________ F_{Quad} = ________________ F_{Cubic} = ________________ $F_{.95,df1,df2}$ = ________________

Q.7. Consider a model with g = 4 treatments, with sample sizes $n_1 = n_2 = n_3 = n_4 = 5$.

p.7.a. Give the rejection region for testing H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

p.7.b. Give the non-centrality parameter for the F-statistic for testing H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$ when $\mu_1 = 50$, $\mu_2 = 60$, $\mu_3 = 40$, $\mu_4 = 50$ and $\sigma = 20$. Give 2Ω

p.7.c. On the following graph, identify the distributions of the F-statistic under H_0 and under the parameter values in p.7.b. Sketch the power of the F-test under p.7.b.