Nested and Crossed/Nested Designs

Q.1. A tire company with 4 factories has 3 teams of workers at each factory (the teams at Factory A are not the same as those at Factory B, and so on). The company is interested in comparing output among factories, as well as among teams within factories. The experiment is conducted as measuring the output of each team on 3 occasions.

p.1.a. Write out the appropriate statistical model, stating all elements (parameters and random variables) and ranges of subscripts.

p.1.b. The following table gives means (SDs) for all teams for all factories. Give the ANOVA table, and test statistics and critical values.

Factory\Team	**1**	**2**	**3**
A	**100 (10)**	**90 (8)**	**110 (6)**
B	**65 (8)**	**70 (6)**	**75 (10)**
C	**110 (8)**	**120 (6)**	**130 (10)**
D	**95 (6)**	**125 (8)**	**110 (10)**

Source	**df**	**SS**	**MS**	**F_0**	**F(.05)**

p.1.c. Use Bonferroni's and Tukey's methods to obtain simultaneous 95% Confidence Intervals among Factory Means.

Q.2. A 2-Factor (nested, fixed effects ANOVA) is fit, with factor A at 4 levels, and factor B at 3 levels (within each level of A). There were 6 replicates for each combination of factor levels. Compute Bonferroni's and Tukey's minimum significant differences for comparing all pairs of means (Factor A) with an experiment-wise error rate of 0.05. Note: This is the same as the margin of error for the point estimates (half-width of simultaneous CI's). Give your results as functions of only MSE. The model is:

$$y_{ijk} = \mu + \alpha_i + \beta_{j(i)} + e_{k(ij)} \quad i = 1,2,3,4 \quad j = 1,2,3 \quad k = 1,\ldots,6$$

$$\sum_{i=1}^{4} \alpha_i = 0 \quad \sum_{j=1}^{3} \beta_{j(i)} = 0 \ \forall i \quad e_{k(ij)} \sim NID\left(0, \sigma^2\right)$$

Bonferroni ____ Tukey ____

Q.3. An experiment is conducted to compare 4 **T**emperatures for cooking bread. A sample of 12 **B**atches of flour are obtained and randomized so that 3 batches were assigned to Temp1, 3 to Temp2, 3 to Temp3, and 3 to Temp4. After cooking, each batch was split into 2 pieces, and each piece was rated on "crustiness." The following table gives the data.

	Temp1			**Temp2**			**Temp3**			**Temp4**	
Batch1	**Batch2**	**Batch3**	**Batch4**	**Batch5**	**Batch6**	**Batch7**	**Batch8**	**Batch9**	**Batch10**	**Batch11**	**Batch12**
22,18	**22,26**	**19,25**	**26,30**	**25,27**	**32,28**	**27,33**	**27,29**	**29,35**	**28,32**	**31,33**	**31,37**

p.3.a. Write out the statistical model.

p.3.b. Obtain the Analysis of Variance

p.3.c. Test H_0: No Temperature effects

p.3.c.i. Test Statistic ______________________________________ p.3.c.ii. Rejection Region _________________________________

p.3.d. Compute the Bonferroni Minimum Significant Difference for comparing all pairs of Temperature means.

Minimum Significant Difference = ____

Q.4. A study is conducted to compare 3 methods of teaching speed reading to adults. A sample of 9 adults (with no prior exposure to speed-reading techniques) is taken, and randomly assigned so that 3 adults receive method 1, 3 receive method 2, and 3 receive method 3. Each adult is observed reading 4 texts of the same length, and *y*, the amount of time needed to complete the text is measured. The model fit is:

$$y_{ij} = \mu + \alpha_i + b_{j(i)} + e_{k(ij)} \quad i = 1,\dots,a \quad j = 1,\dots,b \quad k = 1,\dots,r$$

$$\sum_{i=1}^{a} \alpha_i = 0 \quad b_{j(i)} \sim NID\left(0, \sigma^2_{B(A)}\right) \quad e_{k(ij)} \sim NID\left(0, \sigma^2\right) \quad \{b\} \perp \{e\}$$

The means (SDs) for each adult are given below (in minutes).

Method	1	1	1	2	2	2	3	3	3
Adult	1	2	3	4	5	6	7	8	9
Mean (SD)	120 (10)	125 (12)	115 (8)	105 (12)	110 (10)	115 (8)	135 (10)	130 (12)	125 (8)

Complete the following ANOVA table.

Source	df	SS	MS	F	F(0.05)
Method					
Adult(Method)					
Error					
Total					

Q.5 Researchers conducted an experiment measuring acoustic metric values in *a* = 3 habitats (1=Cliff, 2=Mud, 3=Gravel). Nested within each habitat, there were *b* = 3 random patches. Replicates representing *r* = 5 sites within each patch were obtained. The habitats are considered to be fixed levels, while patches within habitats are considered to be random. The response measured was snap amplitude. The model is given below, numbers in the ANOVA are rounded.

$$y_{ijk} = \mu + \alpha_i + b_{j(i)} + e_{k(ij)} \quad i = 1,\dots,a \quad j = 1,\dots,b \quad k = 1,\dots,r$$

$$\sum_{i=1}^{a} \alpha_i = 0 \quad b_{j(i)} \sim NID\left(0, \sigma^2_P\right) \quad e_{k(ij)} \sim NID\left(0, \sigma^2\right) \quad \{b\} \perp \{e\}$$

p.5.a. Complete the following ANOVA Table and give the expected mean square for each row. Note that for the F-Stats, you are testing: $H_0^H: \alpha_1 = ... = \alpha_a = 0 \quad H_A^H:$ Not all $\alpha_i = 0 \qquad H_0^P: \sigma_P^2 = 0 \quad H_A^J: \sigma_P^2 > 0$

Source	df	SS	MS	F-Stat	F(.05)	E(MS)
Habitat		400.0				
Patch(Hab)		300.0				
Error						
Total		1100.0				

p.5.b Compute Tukey's and Bonferroni's minimum significant difference for comparing pairs of habitat means.

Tukey's HSD: ___________________________________ Bonferroni's MSD: ____________________________

p.5.c Obtain point estimate for σ_P^2 and σ^2

Q.6. A study is conducted to compare pH levels in rivers in the 3 geographic areas of a state. Random samples of 5 rivers were selected within each of the geographic areas, and 4 replicates were obtained within each river.

p.6.a. Write out the statistical model. Be very specific.
p.6.b. Complete the following Analysis of Variance table.

Source	df	SS	MS	F	F(.95)
Area		4000			
River w/in Area		2400			
Error		2250			
Total					

p.6.c. Compute Bonferroni's minimum significant difference for comparing all pairs of geographic areas.

Q.7. A study compared 2 types of fuel sources (carbonized briquettes and charcoal) in Uganda. There were 4 categories nested within each type of fuel source, and 3 replicates for each category within each fuel source. One response reported was Time of Boil (minutes). For this analysis, it seems reasonable to treat both fuel type and category as fixed factors.

p.7.a. Write the statistical model.

p.7.b. The error sum of squares is 144. The means for the 4 categories of carbonized briquettes were: 42, 23, 32, 53. The means for the 4 categories of charcoal were: 20, 19, 25, 24. Compute the Analysis of Variance.

Source	df	Sum of Squares	Mean Square	F	F(0.05)

p.7.c. Obtain a 95% Confidence Interval for the difference between the effects of carbonized briquettes and charcoal.