Block Designs

Part A. Randomized Complete Block Design

QA.1. Jack and Jill wish to compare the effects of 3 interior design presentation methods (dp1, dp2, dp3) on ease of visualizing the task (y = 1 (very difficult) to 7 (very easy)). The sample consisted of 32 participants who had recently remodeled or built a home. Each subject was exposed to each design presentation method, in random order.

The means for the 3 design presentation method are:

$$\overline{y}_{1\bullet} = \frac{188}{32} = 5.875 \quad \overline{y}_{2\bullet} = \frac{218}{32} = 6.813 \quad \overline{y}_{3\bullet} = \frac{199}{32} = 6.219 \quad \overline{y}_{\bullet\bullet} = \frac{605}{96} = 6.302$$

$$SSTR = b\sum_{i=1}^{t}\left(\overline{y}_{i\bullet} - \overline{y}_{\bullet\bullet}\right)^2 = 14.4 \quad SSBL = t\sum_{j=1}^{b}\left(\overline{y}_{\bullet j} - \overline{y}_{\bullet\bullet}\right)^2 = 70.5 \quad TSS = \sum_{i=1}^{t}\sum_{j=1}^{b}\left(\overline{y}_{ij} - \overline{y}_{\bullet\bullet}\right)^2 = 122.5$$

pA.1.a. Jack conducts the analysis as a Completely Randomized Design (independent samples),

Give Jack's test statistic for testing H_0: No design presentation effects:

pA.1.a.i. Test Statistic:

pA.1.a.ii. Reject H_0 if Jack's test statistic falls in the range ____

pA.1.b. Jill conducts the analysis as a Randomized Block Design, treating participants as blocks,

Give Jill's test for testing H_0: No design presentation effects:

pA.1.b.i. Test Statistic:

pA.1.b.ii. Reject H_0 if Jill's test statistic falls in the range ____

pA.1.c. Obtain Jack's and Jill's minimum significant differences based on Tukey's method for comparing all pairs of design presentation effects:

Jack's W_{ij} = _______________________________ Jill's W_{ij} = _________________________________

QA.2. A randomized complete block design is conducted with 3 (fixed) treatment in 3 (random) blocks.

$$y_{ij} = \mu + \tau_i + b_j + e_{ij} \quad i, j = 1,2,3 \quad b_j \sim NID\left(0, \sigma_b^2\right) \quad e_{ij} \sim NID\left(0, \sigma^2\right) \quad \left\{b_j\right\} \perp \left\{e_{ij}\right\}$$

p A.2.a. $E(y_{ij})$ = p A.2.b. $V(y_{ij})$ = p A.2.c. $COV(y_{ij}, y_{i'j})$ = p A.2.d. $COV(y_{ij}, y_{ij'})$ =

p A.2.e. **Derive** $V\left(\overline{y}_{i\bullet}\right), \quad COV\left(\overline{y}_{i\bullet}, \overline{y}_{i'\bullet}\right), \quad V\left(\overline{y}_{i\bullet} - \overline{y}_{i'\bullet}\right)$ **(SHOW ALL WORK)**

QA.3. A randomized block design is conducted to compare t=3 treatments in b=4 blocks. Your advisor gives you the following table of data from the experiment (she was nice enough to compute treatment, block, and overall means for you), where: $TSS = \sum\left(Y-\overline{Y}\right)^2$

Blk\Trt	1	2	3	BlkMean
1	20	22	24	22
2	10	13	16	13
3	28	25	34	29
4	10	12	14	12
TrtMean	17	18	22	19
TSS				
658				

pA.3.a. Complete the following ANOVA table:

Source	df	SS	MS	F_obs	F(.05)	Reject H0: No Effect?
Treatments						
Blocks						
Error						
Total						

pA.3.b. Compute the Relative Efficiency of having used a Randomized Block instead of a Completely Randomized Design

$$RE = \frac{s_{CR}^2}{s_{RBD}^2} = \frac{(r-1)MSB + r(t-1)MSE}{\left(rt-1\right)MSE}$$

RE(RB,CR) = ____________ Sample Size per treatment for equivalent Std. Errors of difference of Means __________

pA.3.c.. Compute Tukey's minimum significant difference for comparing all pairs of treatment means:

Tukey's W = ________________________________

pA.3.d. Give results graphically using lines to connect Trt Means that are not significantly different: T1 T2 T3

QA.4. A randomized complete block design is conducted with 2 (fixed) treatments in 3 blocks.

Case 1: Random Blocks

$$y_{ij} = \mu + \tau_i + b_j + e_{ij} \quad i=1,2;\ j=1,2,3 \quad \tau_1+\tau_2=0 \quad b_j \sim NID\left(0,\sigma_b^2\right) \quad e_{ij} \sim NID\left(0,\sigma^2\right) \quad \left\{b_j\right\} \perp \left\{e_{ij}\right\}$$

pA.4.a. $E(y_{ij})$ =

pA.4.b. $V(y_{ij})$ =

pA.4.c.($i \neq i'$) $COV(y_{ij}, y_{i'j})$ =

pA.4.d. ($j \neq j'$) $COV(y_{ij}, y_{ij'})$ =

pA.4.e. **Derive** $V\left(\overline{y}_{i\bullet}\right), \quad COV\left(\overline{y}_{i\bullet}, \overline{y}_{i'\bullet}\right), \quad V\left(\overline{y}_{i\bullet} - \overline{y}_{i'\bullet}\right)$ **(SHOW ALL WORK)**

Case 2: Fixed Blocks

$$y_{ij} = \mu + \tau_i + \beta_j + e_{ij} \quad i = 1,2;\ j = 1,2,3 \quad \tau_1 + \tau_2 = 0 \quad \beta_1 + \beta_2 + \beta_3 = 0 \quad e_{ij} \sim NID\left(0, \sigma^2\right)$$

pA.4.f. $E(y_{ij}) =$

pA.4.g. $V(y_{ij}) =$

pA.4.h. $(i \neq i')$ $COV(y_{ij}, y_{i'j}) =$

pA.4.i. $(j \neq j')$ $COV(y_{ij}, y_{ij'}) =$

pA.4.j. **Derive** $V\left(\overline{y}_{i\bullet}\right), \quad COV\left(\overline{y}_{i\bullet}, \overline{y}_{i'\bullet}\right), \quad V\left(\overline{y}_{i\bullet} - \overline{y}_{i'\bullet}\right)$ **(SHOW ALL WORK)**

QA.5. Authors of an academic research paper report that they conducted a randomized block design, with 3 treatments and 8 blocks. They report that the Bonferroni Minimum Significant Difference for comparing pairs of treatment means is B = 12.0. What is the Mean Square Error from their Analysis of Variance?

QA.6. Authors of an academic research paper report that they conducted a randomized block design, with 5 treatments and 12 blocks. They report that the Bonferroni Minimum Significant Difference for comparing pairs of treatment means is B = 72.0. What is the Mean Square Error from their Analysis of Variance?

Part B. Latin Square Designs

QB.1. A latin-square design is conducted to compare 5 types of packaging for a food product, in 5 stores, over a 5 week period. The experiment is set up so that each package appears once at each store, and once over each of the 5 weeks.

pB.1.a. Write the statistical model, assuming fixed packages and random stores and weeks, y_{ij} is sales for store i, week j.

pB.1.b. Complete the following Analysis of Variance Table based on the data below:

Store\Week	1	2	3	4	5	St Mean	Pack Mean
1	100	120	110	90	80	100	109
2	140	150	160	145	155	150	92
3	60	40	50	70	30	50	102
4	110	120	130	90	100	110	100
5	90	70	80	110	100	90	97
Wk Mean	100	100	106	101	93	100	100
ANOVA							
Source	**df**	**Sum Sq**	**Mean Sq**	**F**	**F(0.05)**	**Package Effects?**	
Package						**Yes / No**	
Store				**#N/A**	**#N/A**		
Week				**#N/A**	**#N/A**		
Error				**#N/A**	**#N/A**		
Total		**30250**	**#N/A**	**#N/A**	**#N/A**		

pB.1.c. Compute Tukey's and Bonferroni's Minimum significant differences for comparing all pairs of packaging effects:

QB.2. A latin-square design is used to compare 6 brands of energy enhancing drinks on alertness, measured by the amount of items completed in a fixed amount of time on skills tests. There are 6 different skills tests, and 6 participants, such that each energy drink is observed once on each skills test and once for each participant. The drink means and partial ANOVA table are given below.

pB.2.a. Complete the tables.

	Drink1	**Drink2**	**Drink3**	**Drink4**	**Drink5**	**Drink6**	**Overall**
Mean	**30**	**50**	**36**	**42**	**44**	**38**	**40**
SS							

Source	**df**	**SS**	**MS**	**F**	**F(.05)**
Energy Drink					
Skills Test		**460**			
Participant		**2000**			
Error					
Total		**4600**			

pB.2.b. Compute Tukey's HSD and Bonferroni's MSD for comparing all pairs of drinks.

Tukey's HSD: ________________________________ Bonferroni's MSD ____________________________________

QB.3. A latin-square design is used to compare 4 driving conditions (treatments) of speed and road speed/surface condition (Fast/Wet, Fast/Dry, Moderate/Wet, Moderate/Dry) under 4 row gender/passenger-driver groups (MD/FP,MD/MP,FD/FP,FD/MP), and 4 column curve radius levels (16m,26m,60m,100m). **The square was replicated 4 times, with 4 replicates in each cell of the square.**

Grp\Rad	16m	26m	60m	100m
MD/FP	Fast/Dry	Mod/Wet	Mod/Dry	Fast/Wet
MD/MP	Mod/Dry	Fast/Wet	Fast/Dry	Mod/Wet
FD/FP	Fast/Wet	Fast/Dry	Mod/Wet	Mod/Dry
FD/MP	Mod/Wet	Mod/Dry	Fast/Wet	Fast/Dry

pB.3.a. Complete the following ANOVA table.

ANOVA					
Source	df	SS	MS	F	F(0.05)
Trts					
Speed					
Surface					
Spd*Srf					
Rows		110.55			
Cols		6109.32			
Error					
Total		8614.36			

Trt	1 (F/W)	2 (F/D)	3 (M/W)	4 (M/D)	overall
Mean	44.98	45.95	36.25	34.62	40.45

pB.3.b. Compute Tukey's HSD and Bonferroni's MSD for comparing all pairs of treatments (speed/surface combinations).

Tukey's HSD: ______________________________ Bonferroni's MSD __________________________________

QB.4. A study was conducted as a Latin Square design with 5 packages (treatments), in 5 stores (Rows), over 5 weeks (Columns). The response was number of packages sold in a week. The Error sum of squares was reported to be 600. Compute Tukey's HSD and Bonferroni's MSD for comparing all pairs of package means.

QB.5. A bioavailability study for sulpride involved 3 treatments (sulpride alone (A), sulpride with sucralafate (B), and sulpride with antacid (C)). It was conducted in 3 time periods with 6 subjects in a replicated latin square. The ANOVA and the design are given below, for Y = Urinary excretion of sulpride in 24 hours. The means are:

$\overline{Y}_A = 31.0 \quad \overline{Y}_B = 18.6 \quad \overline{Y}_C = 20.9 \quad \overline{Y}_{\bullet\bullet} = 23.5$

pB.5.a. Complete the ANOVA table.

Source	df	SS	MS	F_obs	F(0.05)
Trts					
Periods		51		#N/A	#N/A
Subjects		380		#N/A	#N/A
Error				#N/A	#N/A
Total		1083	#N/A	#N/A	#N/A

Subject\Period	1	2	3
1	A	B	C
2	C	A	B
3	B	C	A
4	C	A	B
5	B	C	A
6	A	B	C

pB.5.b. How many times does each treatment appear in each subject?

pB.5.b. How many times does each treatment appear in each Period?

pB.5.c. Use Tukey's HSD to compare all pairs of treatment means.

HSD_{ij} = ________________________________ Connect trts that are not significantly different B C A

QB.6. An experiment is conducted as a Latin Square with t = 4 treatments. The row factor is random and the column factor is fixed. If squares are replicated, with different levels of the row factor in each square, how many squares will be needed to assure there are at least 50 degrees of freedom for error. Show calculations for each number of squares, 1,2,…

QB.7. A bioequivalence study is conducted to compare 3 formulations of a drug (A = solution, B = tablet, C = capsule) in terms of bioavailability. The study has 3 periods and 3 subjects. Each subject will receive one formulation in each period, and each formulation will appear once in each period. The data are given below.

Period\Subject	1	2	3
1	A 240	B 180	C 300
2	B 216	C 204	A 210
3	C 264	A 192	B 192

Compute the Analysis of Variance for this study. Note that the Total (Corrected) Sum of Squares is 12240.

Source	df	Sum of Squares	Mean Square	F	F(0.05)

Part C: Balanced Incomplete Block Designs

QC.1. A researcher is interested in obtaining preferences among 7 products for consumers. She knows that individuals have a difficult time making comparisons among more than 3 items. Construct a minimal Balanced Incomplete Block Design (with consumers acting as blocks, each comparing 3 products). Give the values: t (number of "treatments"), k (block size), b (the number of blocks), r (number of reps per treatment), λ (the number of consumers comparing any pair of treatments), and the error degrees of freedom. Give the design as well.

Block\Product	1	2	3	4	5	6	7

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t = _________ k = _____________ b = _____________ r = _________ λ = _________ df_E = _____________

QC.2. An experimenter wishes to conduct a taste experiment to determine whether there are differences in the 6 recipes that are to be compared. He plans to have individuals serve as blocks and taste items from the various recipes. However, he knows that individuals have a difficult time choosing a favorite as the number of choices increases. He has selected a sample of 10 raters, and has decided that each will choose his/her favorite among 3 recipes. Complete the following parts, where t = # trts, b=# blocks, r = reps/trt, k = block size, λ = 3 blocks each pair of treatments appear in together.

pC.2.a. t = ____________ b = ____________ r = ____________ k = ____________ λ = ____________

pC.2.b. Put a check in each appropriate cell for this design to meet the criteria in part pC.2.a.

	Recipe1	Recipe2	Recipe3	Recipe4	Recipe5	Recipe6
Rater1						
Rater2						
Rater3						
Rater4						
Rater5						
Rater6						
Rater7						
Rater8						
Rater9						
Rater10						

QC.3. A Balanced Incomplete Block Design is conducted, to compare 7 (fixed) machines in terms of output. The supplier provides (random) batches (blocks) of components that only have 4 components.

$Y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \quad i = 1,\dots,t;\ j = 1,\dots,b$ Note: not all pairs (i, j)

pC.3.a. How many batches will be needed so that each machine produces 4 items? Fill in the following numbers, where t = #trts, k = block size, r = # reps/trt, b = # blocks, λ = # blocks each pair of treatments appears in together.

$t =$ ____________ $k =$ ____________ $r =$ ____________ $b =$ ____________ $\lambda =$ ____________

pC.3.b. The error sum of squares for a model that contains only blocks as a factor is 200. The error sum of squares for a model that contains both blocks and treatments as factors is 120. Test for treatment effects (adjusted for blocks). H_0: $\tau_1 = \dots = \tau_\tau = 0$.

Test Statistic: ____________________________ Rejection Region: ____________________________

pC.3.c. We wish to obtain simultaneous 95% Confidence Intervals for all pairs of treatment differences based on all the intra-block analysis, where: $\hat{V}\left\{\hat{\tau}_i - \hat{\tau}_{i'}\right\} = \dfrac{2kMS_{\text{ERR}}}{\lambda t}$. Give the form of the simultaneous 95% CI's:

$$\left(\hat{\tau}_i - \hat{\tau}_{i'}\right) \pm$$

QB.4. An experiment with g = 4 (fixed) treatments (A,B,C,D) is to be conducted in b = 4 (fixed) blocks of size k = 3.

pB.4.a. Give r (number of replicates per treatment) and λ (the number of blocks that all pairs of treatment appear together in) and fill in one possible design in the following table.

Block\Position	**1**	**2**	**3**
1			
2			
3			
4			

$r =$

$\lambda =$

pB.4.b. Write the statistical model.

pB.4.c. Derive the least squares estimator of μ (overall mean).

pB.4.d. Give $\dfrac{\partial Q}{\partial \alpha_1}$ and solve for $\hat{\alpha}_1$ as a function of other model terms: $\hat{\mu}$, $\left\{\hat{\beta}_j\right\}$, $\left\{n_{ij}\right\}$