Logistic, Poisson, and Nonlinear Regression Problems

Part A: Logistic Regression

QA.1. A study is conducted to measure the effects of levels of a herbicide on the probability of death for 2 weed varieties. The researcher selects 5 dosage levels, and assigns each to 100 weeds of each variety. (Note there are 500 weeds of each variety in study). The following table gives the number of weeds (successfully) eradicated.

		Proportions Dead			Proportions Dead	
Dose	A Dead	A observed	A Fitted	B Dead	B observed	B Fitted
1	5			10		
2	15	#N/A	#N/A	28	#N/A	#N/A
4	32	#N/A	#N/A	52	#N/A	#N/A
8	54	#N/A	#N/A	74	#N/A	#N/A
16	95			100		

Consider the following 3 models (π=P(Death), X_1=Dose, X_2=1 if Weed A, 0 if Weed B):

Model 1 (Dose): $\log\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_1 X_1$

Model 2 (Dose, Variety): $\log\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$

Model 3: (Dose, Variety, Interaction): $\log\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2$

Model	-2logL	B0	B1	B2	B3
1	912.7	-2.15	0.36	#N/A	#N/A
2	885.6	-1.80	0.36	-0.87	#N/A
3	883.3	-1.98	0.42	-0.52	-0.08

Test whether there is a significant interaction between dose and variety (α=0.05).

H_0: H_A:

Test Statistic:

Reject H_0 if the test statistic falls in the range ______________________________

Assuming no significant dose/variety interaction, test for variety effect (controlling for dose). (α=0.05).

H_0: H_A:

Test Statistic:

Reject H_0 if the test statistic falls in the range ______________________________

Give the observed and fitted proportions dead for each variety at doses 1 and 16 based on model 2.

QA.2. A study was conducted to measure the effects of age and motorcycle riding on the incidence of erectile dysfunction (ED). Men were classified by age (20-29,30-39,40-49,and 50-59), where the midpoints (25,35,45, and 55) were used as the **age** levels, and **mtrcycl** was classified as 1 if motorcycle rider and 0 if not. The variable **mtcrage** was obtained by taking the product of **age** and **mtrcycl**. The following models were fit (where π is the probability the man suffers from ED:

$$\text{Model0: } \pi = \frac{e^{\alpha}}{1+e^{\alpha}} \qquad -2\ln(L_0) = 1349.16$$

$$\text{Model1: } \pi(\text{Age, MR}) = \frac{e^{\alpha+\beta_A A+\beta_M M+\beta_{AM} AM}}{1+e^{\alpha+\beta_A A+\beta_M M+\beta_{AM} AM}} \qquad -2\ln(L_1) = 1250.34$$

Model 0:

Variables in the Equation

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 0	Constant	-.269	.064	17.565	1	.000	.764

Model 1:

Variables in the Equation

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1[a]	age	.016	.008	4.076	1	.044	1.017
	mtrcycl	.013	.677	.000	1	.985	1.013
	mtrcage	.039	.018	4.790	1	.029	1.040
	Constant	-1.257	.335	14.102	1	.000	.284

a. Variable(s) entered on step 1: mtrcage.

Test H_0: $\beta_A = \beta_M = \beta_{AM} = 0$ at the $\alpha = 0.05$ significance level:

Test Statistic:

Rejection Region:

Does the "effect" of age differ among motorcycle riders and non-motorcycle riders?

H_0: ______ H_A: _____ TS: __________ P-value ______ **Yes / No**

Give the predicted values for Models 0 and 1 for age=25/mtrcycl=0 and 55/1

Model 0: 25/0 ___________________ 55/1 ________________________

Model 1: 25/0 ___________________ 55/1 ________________________

QA.3. A study was conducted to observe the effect of ginkgo on acute mountain sickness (AMS) in Himalayan trekkers. Trekkers were given either acetozalomide (ACET=1) or placebo (ACET=0) and either ginkgo biloba (Ginkgo=1) or placebo (Ginkgo=0). Further a cross-product term was created: Acetgink=ACET*GINKGO. Three models are fit:

$$\pi(ACET)=\frac{e^{\alpha+\beta_A A}}{1+e^{\alpha+\beta_A A}} \quad \pi(ACET,Ginkgo)=\frac{e^{\alpha+\beta_A A+\beta_G G}}{1+e^{\alpha+\beta_A A+\beta_G G}} \quad \pi(ACET,Ginkgo)=\frac{e^{\alpha+\beta_A A+\beta_G G+\beta_{AG} A*G}}{1+e^{\alpha+\beta_A A+\beta_G G+\beta_{AG} A*G}}$$

Model	Null (No IVs)	Acet	Acet,Ginkgo	A,G,A*G
-2ln(L)	532.378	501.663	501.444	501.318

Variables in the Equation

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1[a]	acet	-1.235	.233	28.085	1	.000	.291
	Constant	-.656	.135	23.542	1	.000	.519

a. Variable(s) entered on step 1: acet.

p.3.a. Test whether there is a significant Acetozalomide Effect :

p.3.a.i. H_0: H_A:

p.3.a.ii. Likelihood-Ratio Test Statistic: ______________________ Rejection Region: _______________

p.3.a.iii. Wald Test Statistic: ________________________________ Rejection Region: ________________

p.3.b. Test whether there is either a Ginkgo main effect and/or ACET*GINKGO interaction (controlling for ACET)

p.3.b.i. H_0: H_A:

p.3.b.ii. Likelihood-Ratio Test Statistic: _____________________ Rejection Region: ___________________

p.3.c. Based on Model 1 give the predicted probabilities of suffering from AMS for the Acetozalomide and non-acetozalomide users:

Acetozalomide:

Non-Acetozalomide:

QA.4. A logistic regression model is fit relating 2-week post-exposure brand recall (Y=1 if Yes, 0 if No) to Exposure to Comedic Violence in advertisement. The Predictors are HI (High-intensity = 1, Low-Intensity = 0), and SC (Severe Consequences = 1, Not Severe = 0). Consider the following 5 Models of the probability that the brand is recalled (π), based on a logit link:

Mod 0: $\ln\left(\frac{\pi}{1-\pi}\right)=\beta_0$ Mod 1: $\ln\left(\frac{\pi}{1-\pi}\right)=\beta_0+\beta_{HI}HI$ Mod 2: $\ln\left(\frac{\pi}{1-\pi}\right)=\beta_0+\beta_{SC}SC$

Mod 3: $\ln\left(\frac{\pi}{1-\pi}\right)=\beta_0+\beta_{HI}HI+\beta_{SC}SC$ Mod 4: $\ln\left(\frac{\pi}{1-\pi}\right)=\beta_0+\beta_{HI}HI+\beta_{SC}SC+\beta_{HI*SC}HI*SC$

-2lnLikelihood for the 5 models are: Mod 0: 185.50 Mod1: 180.50 Mod 2: 182.40 Mod3: 177.36 Mod 4: 175.51

p.4.a. Test whether Probability of Brand Recall is associated with High Intensity (Not controlling for SC):

Test Statistic: ____________________________ Reject H_0 if the test statistic falls in the range _____________________

p.4.b. Test whether there is a significant interaction between HI and SC (controlling for their main effects):

Test Statistic: ____________________________ Reject H_0 if the test statistic falls in the range _____________________

p.4.c. Consider Model 3 (although SC is only moderately significant):

p.4.c.i. Give the predicted probabilities for the four conditions (HI=0/SC=0, HI=1/SC=0, HI=0/SC=1, HI=1/SC=1):

Variables in the Equation

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1[a]	SC	.640	.364	3.087	1	.079	1.897
	HI	.806	.363	4.926	1	.026	2.239
	Constant	-1.431	.343	17.429	1	.000	.239

a. Variable(s) entered on step 1: SC, HI.

HI=0/SC=0: _______________ HI=1/SC=0: ______________ HI=0/SC=1: _______________ HI=1/SC=1: _____________

p.4.c.ii. Construct a 95% CI for the Odds Ratio (HI=1/HI=0), controlling for SC

Lower Bound = ______________________________ Upper Bound = ____________________________________

QA.5. A study was conducted to relate probability of returning to a whale viewing boat tour (Y=1 if Yes, Y=0, if No) to several predictors, based on a sample of n=410 tourists after the tour:

- X_1 = Perceived Crowding (1 if the # of boats affected their enjoyment, 0 if Not)
- X_2 = Reported Crowding (# of boats subject saw near whales)
- X_3 = Subjective Norm (1 if Societal Pressure to do act, such as conservation, 0 if Not)
- X_4 = Income ($1000s/Month)
- X_5 = Prices of Substitute activities (Scuba-Diving, Rafting, and Snorkeling)

p.5.a. Complete the following Table.

Variable	Estimate	Std Err	Lower Bound CI	Upper Bound CI	Odds Ratio	Odds Ratio LB	Odds Ratio UB
Perceived Crowding	**-0.563**	**0.248**					
Reported Crowding	**-0.125**	**0.059**					
Subjective Norm	**0.421**	**0.217**					
Income	**0.540**	**0.329**					
Prices of Substitute	**0.172**	**0.106**					

p.5.b. Give the predicted probabilities of return for the boat tour for the following levels of the independent variables. Note: the authors did not give intercept, so just assume it is 0.

i) $X_1 = 1, X_2 = 6, X_3 = 0, X_4 = 1, X_5=1$

ii) $X_1 = 0, X_2 = 2, X_3 = 1, X_4 = 3, X_5=3$

QA.6. A logistic regression model was fit to relate probability of growth of CRA7152 in apple juice as a function of several predictors, based on a sample of n=74 experimental runs:

- X_1 = pH (Range = 3.5-5.5)
- X_2 = Nisin Concentration (Range = 0-70)
- X_3 = Temperature (Range = 25-50C)
- X_4 = Brix (Range = 11-19)

Model 0: $\log\left(\frac{\pi}{1-\pi}\right) = \beta_0 \quad -2\ln L_0 = 95.95$ Model 1: $\log\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X \quad -2\ln L_1 = 52.33$

p.6.a. Test $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$

Test Statistic: ______________________________ Rejection Region: _______________________ P-value > < 0.05

p.6.b. Complete the following Table.

Variable	Estimate	Std Err	Lower Bound CI	Upper Bound CI	Odds Ratio	Odds Ratio LB	Odds Ratio UB
pH	1.886	0.541					
Nisin	-0.066	0.019					
Temperature	0.110	0.048					
Brix	-0.312	0.143					
Constant	-7.246	3.219			#N/A	#N/A	#N/A

p.6.c. At what values of the independent variables (within the ranges conducted in the experiment) will the predicted probabilities be the highest and lowest? Compute their predicted probabilities.

Highest: pH = ________ , Nisin = ________, Temp = ________ =, Brix = ________

Lowest: pH = ________ , Nisin = ________, Temp = _________ =, Brix = ________

Highest Prob ____________________________________ Lowest Prob __________________________________

QA.7. A study sampled pillars in n = 29 coal mines. Pillars were classified as being either stable or unstable. Two variables were measured on each pillar: strength/stress ratio (s_s) and width/height ratio (w_h). The following models were fit for the probability (π) that a pillar is classified as stable:

Model 0: $\ln\left(\frac{\pi}{1-\pi}\right)=\alpha \qquad -2\ln L_0 = 40.168$

Model 1: $\ln\left(\frac{\pi}{1-\pi}\right)=\alpha+\beta_{ss}s\_s \qquad -2\ln L_1 = 16.282$

Model 2: $\ln\left(\frac{\pi}{1-\pi}\right)=\alpha+\beta_{ss}s\_s+\beta_{wh}w\_h \qquad -2\ln L_2 = 8.810$

Model 3: $\ln\left(\frac{\pi}{1-\pi}\right)=\alpha+\beta_{ss}s\_s+\beta_{wh}w\_h+\beta_{ss\times wh}s\_sw\_h \qquad -2\ln L_3 = 8.072$

p.7.a. Based on the Likelihood Ratio Test, test whether the interaction between strength/stress ratio and width/height ratio is significant.

Null Hypothesis: Alternative Hypothesis:

Test Statistic: ____________________________________ Rejection Region: _________________________

p.7.b. Based on models 1 and 2, test whether the width/height ratio is associated with pillar stability, controlling for strength/stress ratio:

Null Hypothesis: Alternative Hypothesis:

Test Statistic: ____________________________________ Rejection Region: _________________________

p.7.c. The fit for Model 2 is given below. Compute the estimated probability that a pillar is stable for the following 2 mines: Bellampali: s_s = 2.40 w_h = 1.80 Kankanee: s_s = 0.86 w_h = 2.21

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Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  -13.146      5.184  -2.536   0.0112 *
w_h            2.774      1.477   1.878   0.0604 .
s_s            5.668      2.642   2.145   0.0319 *
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Bellampali: ______________________________ Kankanee: _______________________________

QA.8. A logistic regression model is fit, relating the probability of a product discount coupon being redeemed to the value of the coupon. The coupon values used in the experiment were 25, 50, 75, and 100 cents for a product that costs 250 cents. The regression coefficient for the coupon value is 0.07. This means that as the coupon value increases by 1 cent, the probability the coupon is redeemed increases by 0.07.

True / **False**

QA.9. A study was conducted in France to study the effects of a waitress wearing a red shirt on whether or not a customer leaves a tip. The experiment had waitresses wearing red, white, black, blue, green, and yellow shirts. Also observed was whether the customer was male or female. The response was Y=1 if waitress received a tip, 0 if not. Let X_1 = 1 if shirt was red, 0 if not and X_2 = 1 if customer was male, 0 if female. The models fit were (where π is the probability of a tip):

Model 1: $\ln\left(\frac{\pi}{1-\pi}\right)=\beta_0+\beta_1X_1$ Model 2: $\ln\left(\frac{\pi}{1-\pi}\right)=\beta_0+\beta_1X_1+\beta_2X_2$ Model 3: $\ln\left(\frac{\pi}{1-\pi}\right)=\beta_0+\beta_1X_1+\beta_2X_2+\beta_3X_1X_2$

	Model 1		Model2		Model 3	
Variable	Estimate	StdError	Estimate	StdError	Estimate	StdError
Intercept	-0.5681	0.085	-0.7522	0.1276	-0.634	0.1326
X1 (Red)	0.3708	0.2008	0.3779	0.2014	-0.2956	0.3325
X2 (Male)	#N/A	#N/A	0.3108	0.1575	0.1124	0.1728
X1X2	#N/A	#N/A	#N/A	#N/A	1.1387	0.427
-2lnL	67.26		63.36		55.94	

p.9.a. For model 1, test whether the probability of a consumer gives a tip is different when the waitress wears a red shirt rather than one of the other colors. $H_0: \beta_1 = 0$ $H_A: \beta_1 \neq 0$

Test Statistic ________________________________ Reject Region ___________________________

p.9.b. Based on models 1 and 3, test whether is either a gender main effect and/or gender/red shirt interaction.
$H_0: \beta_2 = \beta_3 = 0$ $H_A: \beta_2$ and/or $\beta_3 \neq 0$

Test Statistic ________________________________ Reject Region ___________________________

p.9.c. For model 3, give the predicted probabilities for each of the following combinations:

Non-Red/Female ____________ Red/Female ____________ Non-Red/Male ____________ Red/Male ____________

QA.10. A study was conducted, relating presence/absence of luxury goods purchasing tendency to horoscope sign

(X_1=1 if Aries, X_2=1 if Taurus, X_3=1 if Gemini, X_4=1 if Cancer, X_5=1 if Leo, X_6=1 if Virgo, X_7=1 if Libra, X_8=1 if Scorpio, X_9=1 if Sagittarius, X_{10}=1 if Capricorn, X_{11}=1 if Aquarius, Pisces is the "reference sign".)

Two models were fit: the null model (intercept only) and the full model with intercept and $X_1,\ldots,X_{11}$.

Null Model: $\log\left(\frac{\pi}{1-\pi}\right)=\beta_0$

```
Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  0.47608    0.09943   4.788 1.68e-06 ***

> logLik(mod1)
'log Lik.' -31.6021 (df=1)
```

Full Model: $\log\left(\frac{\pi}{1-\pi}\right)=\beta_0+\beta_1X_1+...+\beta_{11}X_{11}$

```
Coefficients:
               Estimate Std. Error z value Pr(>|z|)
(Intercept)   -0.051293   0.320362  -0.160   0.8728
X1             0.258933   0.491991   0.526   0.5987
X2             0.543770   0.499095   1.090   0.2759
X3             0.184825   0.438084   0.422   0.6731
X4             0.872274   0.483260   1.805   0.0711 .
X5             0.908744   0.452065   2.010   0.0444 *
X6            -0.002774   0.459150  -0.006   0.9952
X7             1.149906   0.500779   2.296   0.0217 *
X8             0.581922   0.511327   1.138   0.2551
X9             0.510826   0.488463   1.046   0.2957
X10            0.396134   0.450690   0.879   0.3794
X11            1.283437   0.535869   2.395   0.0166 *

> logLik(mod2)
'log Lik.' -23.55665 (df=12)
```

p.10.a. Give the fitted probabilities for people with sign Aries under each model.

Null Model ______________________________ Full Model _____________________________

p.10.b. Test whether there is evidence that luxury goods purchase tendencies differ by horoscope sign.

H_0:

Test Statistic ____________________________ Rejection Region: ____________________ P > or < .05

QA.11. A packaging study was conducted to determine the effects of 3 factors: Product (1=Bottled Water (X_1=0), 2=Carbonated soda can (X_1=1)), Pattern (1=Column (X_2=X_3=0), 2=Interlocking (X_2=1,X_3=0), 3=Pinwheel (X_2=0,X_3=1)), and Speed (X_4= 1.62,8.05,16.09 km/hr)) on the response of whether or not a radio frequency id is scanned on a case of the product on a pallet. The models fit were (where π is the probability of a successful scan):

Model 1: $\ln\left(\frac{\pi}{1-\pi}\right)=\beta_0+\beta_1X_1$ Model 2: $\ln\left(\frac{\pi}{1-\pi}\right)=\beta_0+\beta_1X_1+\beta_2X_2+\beta_3X_3$

Model 3: $\ln\left(\frac{\pi}{1-\pi}\right)=\ln\left(\frac{\pi}{1-\pi}\right)=\beta_0+\beta_1X_1+\beta_2X_2+\beta_3X_3+\beta_4X_4$

	Model1		Model2		Model3	
Variable	Estimate	StdError	Estimate	StdError	Estimate	StdError
Intercept	1.002	0.058	0.806	0.069	1.278	0.081
X1	-2.009	0.065	-2.004	0.065	-2.049	0.066
X2	#N/A	#N/A	0.336	0.063	0.342	0.064
X3	#N/A	#N/A	0.234	0.064	0.238	0.065
X4	#N/A	#N/A	#N/A	#N/A	-0.053	0.005
lnL	-470.7		-455.8		-385.1	

p.11.a. For model 1, test whether the probability of a successful scan is different when the the package is carbonated soda cans than bottled water. H_0: $\beta_1 = 0$ H_A: $\beta_1 \neq 0$

Test Statistic ________________________________ Reject Region ___________________________

p.11.b. Based on models 1 and 2, test whether there are differences among pallet patterns, controlling for product.
H_0: $\beta_2 = \beta_3 = 0$ H_A: β_2 and/or $\beta_3 \neq 0$

Test Statistic ________________________________ Reject Region ___________________________

p.11.c. For model 3, give the predicted probabilities for each of the following combinations:

Soda/Column/16.09 ________________ Water/Interlocking/1.62 ________________

Part B: Poisson Regression

QB.1. A researcher for the National Park Service is interested in the relationship between the density of bears in national parks, and physical characteristics of the parks. Her response is the (estimated via satellite imaging) number of bears (Y) per 100 mi^2 (A), and her predictor variables are the average annual temperature (X_1=degrees F), density of foliage (X_2=% coverage), human population density surrounding park (X_3,=residents/mi^2), and an indicator of whether the park is in a mountainous region (X_4). She fits the Poisson regression model (with log(A) as offset):

$$\log\left(E\left(\frac{Y}{A}\right)\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4$$

She fits the model using a statistical computing package. Under the null model (with all regression coefficients set to 0, she obtains $-2\log L_0=8400$. Under the full model (containing all 4 predictors), she obtains $-2\log L_1=7800$. Test whether there is an association between bear density and any of these 4 independent variables at the $\alpha=0.05$ significance level:

H_0: H_A:

Test Statistic:

Reject H_0 if the test statistic falls in the range ___________________________

The estimated regression coefficients are: $B_0=5$, $B_1=-2.0$, $B_2=3.0$, $B_3=-1.0$, $B_4=5$

Give the predicted number for a park with $A=70$, $X_1=70$, $X_2=15$, $X_3=8$, $X_4=0$

QB.2. A substance is used in biomedical research and shipped by airfreight in cartons of 1000 ampules. Data from n=10 were collected where X = the number of aircraft transfers (0,1,2,3) and Y = the number of broken ampules. A Poisson regression model was fit where the (natural) log of the expected number of broken ampules is linearly related to the number of transfers:

$\ln(\mu) = \beta_0 + \beta_1 X$ $\quad \hat{\beta}_0 = 2.341 \quad SE\left(\hat{\beta}_0\right) = 0.1338 \quad \hat{\beta}_1 = 0.215 \quad SE\left(\hat{\beta}_1\right) = 0.0828$

Test whether there is a positive association between the number of broken ampules and the number of transfers using the Wald "z-test" with α=0.05.

$H_0: \beta_1 = 0 \quad H_A: \beta_1 > 0$

Test Stat: ________________________ Rejection Region: __________________

Give the estimated means for X=0,1,2 transfers:

QB.3. . A study considered the relationship between number of matings (Y) and age (X) among n=41 African elephants. The researchers considered 3 Poisson Regression models with log link functions:

Model 1: $\mu = e^{\beta_0}$ Model 2: $\mu = e^{\beta_0 + \beta_1 X}$ Model 3: $\mu = e^{\beta_0 + \beta_1 X + \beta_2 X^2}$

The results for the 3 models are given below:

Model	B0 (SE)	B1 (SE)	B2 (SE)	-2log(L)
1	.987 (.095)	#N/A	#N/A	2.88
2	-1.582 (.545)	.069 (.014)	#N/A	-20.48
3	-2.857 (3.036)	.136 (.158)	-.001 (.002)	-21.67

p.3.a. Based on Model 2 versus Model 1, test whether there is a (linear) association between the log of the mean number of matings and age, based on the Wald and Likelihood-Ratio Tests. $H_0: \beta_1 = 0 \quad H_A: \beta_1 \neq 0$.

Wald Test Stat: _____________________ LR Test Stat: ____________________ Rejection Region: _____________

p.3.b. Based on Model 3 versus Model 2, test whether there is a nonlinear association between the log of the mean number of matings and age, based on the Wald and Likelihood-Ratio Tests. $H_0: \beta_2 = 0 \quad H_A: \beta_2 \neq 0$.

Wald Test Stat: _____________________ LR Test Stat: ____________________ Rejection Region: _____________

p.3.c. Obtain the predicted value (estimated mean) for elephants of age = 40, based on each model:

Model 1 __________________ Model 2 _________________ Model 3 ________________________

QB4. A Poisson regression model was fit, relating apprentice migration to Edinburgh, from n=33 counties in Scotland during the late 18th century. The response was number of apprentices emigrating to Edinburgh, with predictors: counties' **D**istance, **P**opulation (1000s), degree of **U**rbanization, and direction from Edinburgh (1=North, 2=**W**est, 3=**S**outh). The following regression models were fit, with a log link function, and the reference direction being North:

Model 1: $\ln\left(E\{Y_i\}\right) = \beta_0 + \beta_D D_i + \beta_P P_i + \beta_U U_i + \beta_W W_i + \beta_S S_i$

Model 2: $\ln\left(E\{Y_i\}\right) = \beta_0 + \beta_D D_i + \beta_P P_i + \beta_U U_i + \beta_W W_i + \beta_S S_i + \beta_{DW} D_i W_i + \beta_{DS} D_i S_i$

Model1	Estimate	Std. Error	z value	Pr(>|z|)		Model2	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	4.2517	0.2477	17.164	0.0000		(Intercept)	2.6785	0.2745	9.757	0.0000
Dist	-0.0340	0.0019	-17.592	0.0000		Dist	-0.0115	0.0017	-6.589	0.0000
Pop	0.0213	0.0015	14.014	0.0000		Pop	0.0159	0.0016	9.992	0.0000
Urban	-0.0358	0.0041	-8.837	0.0000		Urban	-0.0188	0.0043	-4.422	0.0000
West	0.2324	0.1836	1.265	0.2060		West	1.6897	0.3652	4.627	0.0000
South	1.1065	0.1500	7.377	0.0000		South	3.7072	0.2525	14.681	0.0000
						Dist*West	-0.0275	0.0057	-4.844	0.0000
						Dist*South	-0.0609	0.0057	-10.74	0.0000
Residual Dev	df					Residual Dev	df			
256.31	27					84.362	25			

Note, that in R, Residual Deviance is ((-2logLikelihood(Current Model)) – (-2logLikelihood(Model with Mean=Y)))

p.4.a. Use the likelihood-ratio test to test H_0: $\beta_{DW} = \beta_{DS} = 0$ (No interaction between Direction and distance).

Test Statistic: ____________________________ Reject H_0 if the test statistic falls in the range _____________________

p.4.b. East Lothian is a distance of 33 from Edinburgh, has a population of 30 (in 1000s), has an Urbanization level of 43.4, and is South of Edinburgh. Give their predicted values for each model, and residuals. The observed number of apprentices is 44.

Model 1: Predicted = __________________________ Residual = _____________________________________

Model 2: Predicted = __________________________ Residual = _____________________________________

QB.5. Three Poisson regression models relating the rate of fatalities for the British rail system versus year (1967-2003) were fit. The rate was (fatalities/million miles of train service). The British railway system was privatized in 1994, and an indicator variable for privatized was created. The following models were fit (where μ_i / t_i = mean rate of fatalities, X_1 = Year – 1967, X_2 = 1 if privatized (post 1994), 0 if not):

1. $\ln\left(\frac{\mu_i}{t_i}\right) = \beta_0 + \beta_1 X_1 \qquad -2\ln L_1 = 486.64$

2. $\ln\left(\frac{\mu_i}{t_i}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \qquad -2\ln L_2 = 443.86$

3. $\ln\left(\frac{\mu_i}{t_i}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 \qquad -2\ln L_3 = 443.36$

Parameter Estimates and Standard Errors are given below for each model.

	Model1	Model2	Model3
Parameter	**Estimate (SE)**	**Estimate (SE)**	**Estimate (SE)**
Intercept	**-1.9069 (0.0431)**	**-2.0430 (0.0492)**	**-2.0469 (0.0496)**
X1	**-0.0209 (0.0022)**	**-0.0058 (0.0032)**	**-0.0055 (0.0032)**
X2	**#N/A**	**-0.5932 (0.0913)**	**-0.0462 (0.7803)**
X1*X2	**#N/A**	**#N/A**	**-0.0172 (0.0245)**

p.5.a. Test H_0: $\beta_2 = \beta_3 = 0$ versus H_A: β_2 and/or $\beta_3 \neq 0$

Test Statistic: ______________________________________ Rejection Region: ______________________________

p.5.b. Assuming the interaction is not significant, Use the Wald Test, and the Likelihood Ratio tests to test for a privatization effect: H_0: $\beta_2 = 0$ versus H_A: $\beta_2 \neq 0$

Wald Statistic: __________________ LR Statistic: ______________________ Rejection Region: _____________

p.5.c. Based on model 2, the fitted values for 1993 (X_1=1993-1967=26, X_2=0, t_{1993}=425) and for 1995 (28, 1, 423) are:

Y-hat(1993)= _______________________________ Y-hat(1995) = _______________________________

QB.6. Three Poisson regression models relating the number of Bigfoot sightings for each state to square root of the state's Wilderness area (X_1, sqrt(Area/1000)) and the state's population (X_2, in millions). The three models fit are:

1. $\ln(\mu_i) = \beta_0 + \beta_1 X_1 \quad -2\ln L_1 = 5368.02$ 2. $\ln(\mu_i) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \quad -2\ln L_2 = 3955.47$
3. $\ln(\mu_i) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 \quad -2\ln L_3 = 3948.26$

Parameter Estimates and Standard Errors are given below for each model.

	Model1	Model2	Model3
Parameter	**Estimate (SE)**	**Estimate (SE)**	**Estimate (SE)**
Intercept	**4.0712 (0.0197)**	**3.6495 (0.0237)**	**3.596 (0.0311)**
X1	**0.2238 (.0078)**	**0.1583 (.0101)**	**0.1777 (0.0121)**
X2	**#N/A**	**0.0577 (.0014)**	**0.0648 (0.0030)**
X1*X2	**#N/A**	**#N/A**	**-0.0023 (0.00084)**

p.6.a. Test H_0: $\beta_2 = \beta_3 = 0$ versus H_A: β_2 and/or $\beta_3 \neq 0$

Test Statistic: ______________________________________ Rejection Region: ______________________________

p.6.b. Ignoring potential interaction, based on Models 2 and 1, Use the Wald Test, and the Likelihood Ratio tests to test for a population effect: H_0: $\beta_2 = 0$ versus H_A: $\beta_2 \neq 0$

Wald Statistic: __________________ LR Statistic: ______________________ Rejection Region: _____________

p.6.c. Based on model 3, the fitted values for Oregon (X_1 = 1.574, X_2 = 3.83) and Florida (X_1 = 1.193, X_2 = 18.80) are:

Y-hat (Oregon)= _______________________________ Y-hat (Florida) = _______________________________

QB.7. A study in Edmonton, Canada modelled the relationship between the number of fresh food stores (including: supermarket, local grocery store, community garden, and farmers' market) in n = 247 shopping districts, with the following independent variables: (percent children, percent seniors, percent unemployed, percent minority, percent with private motor vehicle, percent using public transportation, percent walk, percent bike).

p.7.a. Complete the following table.

Variable	Coefficient	Std. Error	Chi-Square	P > 0.05 or < 0.05	Median
Constant	1.538	0.5			#N/A
Children	-3.787	0.979			23.43
Senior	0.699	0.672			10.91
Unemployment	16.08	2.931			2.13
Minority	0.694	0.428			23.52
Private Vehicle	-1.342	0.61			42
Public Transport	2.903	1.26			6.82
Bicycle	2.954	3.098			0
Walk	5.626	1.583			1.19

p.7.b. The authors used a log link function for the model. Give the predicted number of fresh food stores for a hypothetical shopping district that has the median percentage for each of the independent variables.

QB.8. A study was conducted to determine whether there is an association between a nation's gender equity, and Olympic medal success. There were n = 121 countries used in the analysis. Predictors included: Latitude (X_1, 0=South Pole, 180=North Pole), Gross Domestic Product (X_2), Population (X_3), Gini Index (X_4, higher scores mean higher income inequality), and Gender Gap score (X_5, higher scores mean higher gender equality). The response was the count of medals won in the combined 2014 Winter and 2012 Summer Olympics. Poisson Regression models were fit separately for female and male athletes.

Model: $\ln(\mu) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5$

	Women		Men	
Variable	Estimate	StdError	Estimate	StdError
Intercept	1.01	0.17	1.18	0.16
Latitude	-0.06	0.17	0.00	0.17
GDP	0.39	0.05	0.36	0.05
Population	0.20	0.06	0.14	0.07
Gini Index	-0.46	0.20	-0.66	0.21
Gender Gap Score	0.44	0.15	0.31	0.13

p.8.a. Test whether there is an association between gender gap score and Olympic medal count, controlling for all other predictors.

Females: Test Statistic ____________ Rejection Region ____________ Males: Test Statistic ____________

p.8.b. Test whether there is an association between Gini Index (income inequality) and Olympic medal count, controlling for all other predictors.

Females: Test Statistic ____________ Rejection Region ____________ Males: Test Statistic ____________

p.8.c. How would you interpret these results?

QB.9. A Poisson Regression model relating the number of Caution Flags (crashes) to the number of **L**aps and the **T**rack length for a random sample of n = 60 Nascar races during the 1997-2003 seasons was fit. The models fit and estimates/log-likelihood statistics are given below.

Model 1: $\log(\mu) = \beta_0 \qquad \log(\hat{\mu}) = 2.0450 \qquad l_1 = -168.46$

Model 2: $\log(\mu) = \beta_0 + \beta_L L \qquad \log(\hat{\mu}) = 1.2205 + 0.0025L \qquad l_2 = -144.85$

Model 3: $\log(\mu) = \beta_0 + \beta_T T \qquad \log(\hat{\mu}) = 2.5547 - 0.3700T \qquad l_3 = -152.91$

Model 4: $\log(\mu) = \beta_0 + \beta_L L + \beta_T T \qquad \log(\hat{\mu}) = 0.3321 + 0.0040L + 0.2907T \qquad l_4 = -143.32$

p.9.a. Test whether mean number of crashes is associated with the number of Laps, NOT controlling for Track length.

H_0:

Test Statistic __________________________ Rejection Region: ___________________ P > or < .05

p.9.b. Test whether mean number of crashes is associated with Track length, NOT controlling for the number of Laps.

H_0:

Test Statistic __________________________ Rejection Region: ___________________ P > or < .05

p.9.c. Test whether mean number of crashes is associated with Track length, CONTROLLING for the number of Laps,.

H_0:

Test Statistic __________________________ Rejection Region: ___________________ P > or < .05

p.9.d. The Daytona 500 has 200 Laps on a Track length of 2.50 miles. Give its predicted number of crashes based on Model 4.

p.9.e. What may explain the difference in your results in parts p.9.c. and p.9.d. ______________________________

QB.10. A study in Northern Switzerland fit a Poisson Regression model relating the number of hail days in a month (Y) to the following predictors: Two Meter Temperature (X_1), log Mixed Layer Convective Available Potential Energy (X_2), Wind Shear (X_3), and Dummy Variables for months May-September ($X_4,\ldots,X_8$, with April as reference month).

Model: $\ln(\mu) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_7 X_7 + \beta_8 X_8$

Parameter	Estimate	StdErr
Intercept	-0.748	0.347
X1	0.136	0.034
X2	0.248	0.098
X3	0.032	0.022
X4(May)	1.509	0.318
X5(June)	1.965	0.308
X6(July)	2.965	0.432
X7(Aug)	1.767	0.312
X8(Sept)	0.837	0.341

p.10.a. Test whether there is an association between Wind Shear and Hail Days, controlling for all other predictors.

Test Statistic ______________________________ Rejection Region ______________________ P > or < .05

p.10.b. Give the point estimate and 95% Confidence Interval for the ratio of number of Hail Days in July to April, controlling for all other predictors.

Point Estimate _______________________ 95% CI: ____

Part C: Nonlinear Regression

QC.1. A study was conducted to measure the effects of pea density (X_1, in plants/m^2) and volunteer barley density (X_2, in plants/m^2) on pea seed yield (Y). The researcher fit a nonlinear regression model:

$$E(Y) = \frac{\beta_1 X_1}{1 + \beta_2 X_1 + \beta_3 X_2}$$

Assuming $\beta_1, \beta_2, \beta_3 > 0$, what is E(Y) as volunteer barley density goes to infinity?

The following table gives the estimated regression coefficients, standard errors, z-stats, and P-values (the sample size was huge):

Coefficient	Estimate	Std Error	Z	P-value
B1	7.6	2	3.80	0.00014
B2	0.019	0.007	2.71	0.00664
B3	0.17	0.05	3.40	0.00067

What can we say about mean pea seed yield as volunteer barley density increases, controlling for pea density (α=0.05)?

a) Increases b) Decreases c) Not Related to barley density

What can we say about mean pea seed yield as pea density increases, controlling for volunteer barley density (α=0.05)?

a) Increases b) Decreases c) Not Related to pea density

Give the estimated pea yield for the following combinations of pea density and barley density: (pd=100,bd= 0), (100, 200), (200, 0), (200,200) and plot them on following graph.

100 , 0:

100 , 200:

200 , 0:

200 , 200:

QC.2. A study is conducted to measure the relationship between breaking strength of concrete (Y) and the amounts of 2 key ingredients: A (X_1) and B (X_2). The relationship is believed to be nonlinear, and of the form: $E(Y) = \beta_0 X_1^{\beta_1} X_2^{\beta_2}$. The engineer transforms the model by taking (natural) logarithms on each side to obtain the estimated regression coefficients. The transformed model is: $Y' = b_0' + b_1{}'X_1{}' + b_2 X_2{}'$. She obtains the following estimated regression coefficients: b_0'=6.0, b_1'=0.7, b_2'=0.2. Give the fitted values for the following combinations of X_1 and X_2: (1,1), (1,10), (10,1), (10,10).

(Hints: log(ab)=log(a)+log(b), log(a^b)=b*log(a)).

X_1=1, X_2=1: X_1=1, X_2=10: X_1=10, X_2=1: X_1=10, X_2=10:

QC.3. A model is fit by a mining engineer to relate the angles of subsidence of excavation sights (Y) to the ratio of the width to the depth of the mine (X, ranging from 0.34 to 2.17). She fits the following model based on Mitcherlich's Law of Diminishing marginal Returns based on n=16 wells:

$$y_i = \beta_0[1-\exp(-\beta_1 x_i)] + \varepsilon_i \qquad \hat{\beta}_0 = 32.46 \quad SE\left(\hat{\beta}_0\right) = 2.65 \quad \hat{\beta}_1 = 1.51 \quad SE\left(\hat{\beta}_1\right) = 0.30$$

The first well had x_1=1.11 and y_1=33.6. Give its predicted value and residual:

Predicted: ____________________ Residual __________________________

Compute a 95% Confidence Interval for the maximum mean angle (Hint, use the t-distribution for the critical value):

QC.4. A nonlinear regression model is to be fit, relating Area (Y, in m^2) of palm trees to age (X, in years) by the Gompertz model: E(Y) = α+exp[-β*exp(-γX)] for $\alpha > 0, \beta > 0, \gamma > 0$.

p.4.a. What is E(Y), in terms of the model parameters when X=0?

p.4.b. What is E(Y), in terms of the model parameters when X $\rightarrow\infty$?

QC.5. An enzyme kinetics study of the velocity of reaction (Y) is expected to be related to the concentration of the chemical (X) by the following model (based on n=18 observations):

$$Y_i = \frac{\beta_0 X_i}{\beta_1 + X_i} + \varepsilon_i \quad \varepsilon_i \sim N\left(0, \sigma^2\right)$$

The following results are obtained.

```
                              The NLIN Procedure
                                       Approx
       Parameter       Estimate     Std Error
       b0               28.1          0.73
       b1               12.6          0.76
```

p.5.a. Give a 95% Confidence Interval for the Maximum Velocity of Reaction

p.5.b. Give a 95% Confidence Interval for the dose needed to reach 50% of Maximum Velocity of Reaction

p.5.c. Give the predicted velocity when X=0, 10, 20, 30 and difference between each

$Y_0 =$

$Y_{10} =$

$Y_{20} =$

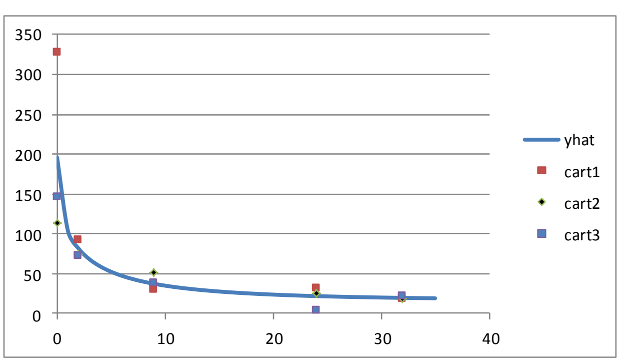
$Y_{30} =$

$Y_{10} - Y_0 =$ $\quad Y_{20} - Y_{10} =$ $\quad Y_{30} - Y_{20} =$

QC.6. A study was conducted, relating Napthelene peak area (Y), measured as a function of the time since discharge (X) of gunshot cartridges. Three cartridges were shot, each measured at X=0,2,9,24,32 hours. Note: the model has serious non-constant variance, ignore this for this problem. The equation fit, based on diffusion theory is:

$$E\{Y\} = \beta_0 + \beta_1 \exp\left\{-\beta_2 \sqrt{X}\right\} \qquad \beta_0 \geq 0, \quad \beta_1 > 0, \quad \beta_2 > 0$$

p.6.a. Give the expected value when: X = 0 ____________________ X→∞ ________________________



```
Formula: y ~ b0 + b1 * exp(-b2 * sqrt(x))
Parameters:
   Estimate Std. Error t value Pr(>|t|)
b0  16.6466    24.5650   0.678  0.51085
b1 178.9799    35.0639   5.104  0.00026 ***
b2   0.7201     0.3923   1.836  0.09127 .
```

p.6.b. Give the fitted values for the following times: t=0, t=10, t=20, t=30 and locate them on the graph.

$\hat{Y}_0 =$ ______________ $\hat{Y}_{10} =$ ______________ $\hat{Y}_{20} =$ ______________ $\hat{Y}_{30} =$ ______________

QC.7. A nonlinear regression model was fit, relating beer foam height (Y, centimeters) to time since pouring (X, seconds) for three brands of beer. We have 3 dummy variables, one for each brand due to the nonlinearity. The model fit is:

$$Z_{1i} = \begin{cases} 1 \text{ if Brand 1 for observation } i \\ 0 \quad \text{otherwise} \end{cases} \qquad Z_{2i} = \begin{cases} 1 \text{ if Brand 2 for observation } i \\ 0 \quad \text{otherwise} \end{cases} \qquad Z_{3i} = \begin{cases} 1 \text{ if Brand 3 for observation } i \\ 0 \quad \text{otherwise} \end{cases}$$

Model 1: (Brand Specific Equations): $Y_i = \beta_{01} Z_{1i} \exp\{-\beta_{11} X Z_{1i}\} + \beta_{02} Z_{2i} \exp\{-\beta_{12} X Z_{2i}\} + \beta_{03} Z_{3i} \exp\{-\beta_{13} X Z_{3i}\} + \varepsilon_i$

Model 2: (Common Equations): $Y_i = \beta_0 \exp\{-\beta_1 X\} + \varepsilon_i = \beta_0 (Z_{1i} + Z_{2i} + Z_{3i}) \exp\{-\beta_1 X (Z_{1i} + Z_{2i} + Z_{3i})\} + \varepsilon_i$

Model 3: (Brand 1 vs Common 2&3): $Y_i = \beta_{01} Z_{1i} \exp\{-\beta_{11} X Z_{1i}\} + \beta_{023} (Z_{2i} + Z_{3i}) \exp\{-\beta_{123} X (Z_{2i} + Z_{3i})\} + \varepsilon_i$

Note: Each brand was observed at 15 time points between 0 and 360 seconds, thus the overall sample size is n=45.

Results for the 3 models are given below.

Model1 (Brand Specific Curves)			Model2 (Common Curves)			Model 3 (Brands 2&3 vs 1)		
Parameter	Estimate	t	Parameter	Estimate	t	Parameter	Estimate	t
b01	16.5	79.3	b0	14.3	20.8	b01	16.5	62.2
b11	0.0034	29.0	b1	0.0049	9.2	b023	0.0034	22.7
b02	13.23	53.6				b11	13.29	61.3
b12	0.0068	26.7				b123	0.0061	29.5
b03	13.37	57.0						
b13	0.0056	26.6						
SSE1	6.03		SSE2	184.09		SSE3	10.31	

p.7.a. Give the fitted values (predicted beer foam height) based on model 1 for each brand at X=120 seconds.

Brand 1 _________________________ Brand 2 __________________________ Brand 3 _________________________

p.7.b. Use Models 1 and 2 to test H_0: $\beta_{01} = \beta_{02} = \beta_{03}$ & $\beta_{11} = \beta_{12} = \beta_{13}$ (Common Curves for All Brands)

Test Statistic: ________________________________ Rejection Region: _____________________________

p.7.c. Use Models 1 and 3 to test H_0: $\beta_{02} = \beta_{03}$ & $\beta_{12} = \beta_{13}$ (Common Curves for Brands 2 and 3)

Test Statistic: ________________________________ Rejection Region: _____________________________

QC.8. A nonlinear regression model was fit, relating cumulative cellular phones in Greece (Y, in millions) to year since 1994 (X=Year-1994). The authors considered various models, including the following Gompertz model:

$E\{Y\} = \beta_0 e^{-e^{-\beta_1 - \beta_2 X}} \quad \beta_0, \beta_2 > 0 \quad e = 2.718...$

```
Formula: phones.m ~ b0 * exp(-exp(-b1 - b2 * t))

Parameters:
   Estimate Std. Error t value Pr(>|t|)
b0 13.37448    0.29214   45.78 5.66e-12 ***
b1 -2.20756    0.08342  -26.46 7.60e-10 ***
b2  0.40323    0.01874   21.52 4.76e-09 ***
```

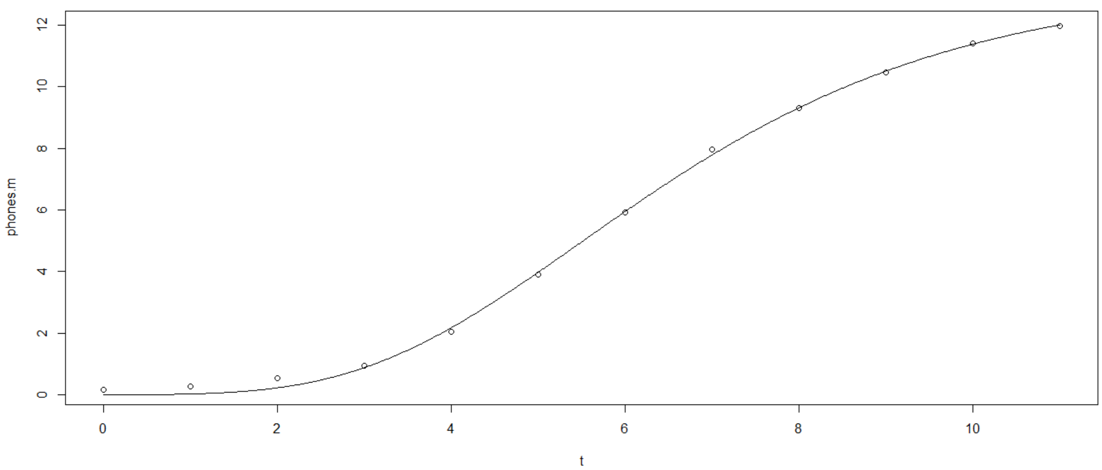
p.8.a. Give the fitted values (predicted cumulative sales) based on the model for years 1994 and 2004.

1994 ______________________________ 2004 ______________________________

p.8.b. β_0 represents the asymptote (maximum total cumulative sales). Obtain a 95% Confidence Interval for β_0.

Lower Bound ____________________ Upper Bound ______________________________

p.8.c. The following plot shows the fitted equation and data. Give an approximate time when sales cross 6 (million units sold).



QC.9. A nonlinear regression model was fit, relating Cutter Life Index (Y) to the quartz content of rocks being cut (X, in %) in tunneling operations. The model fit is:

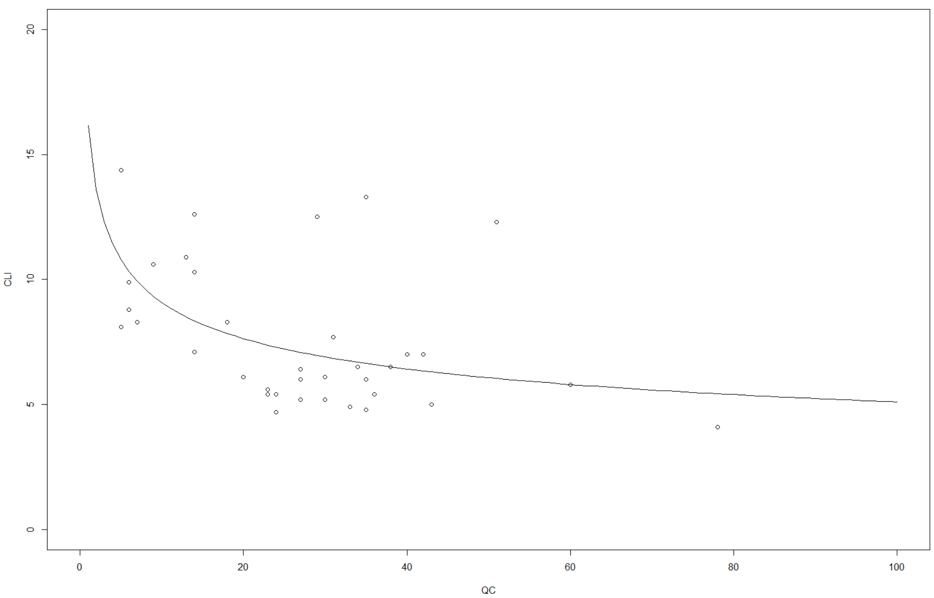
$Y = \beta_0 X^{\beta_1} + \varepsilon \quad \text{with} \;\; \beta_0 > 0 \text{ and } \beta_1 < 0 \quad \text{and} \;\; \varepsilon \sim N\left(0, \sigma^2\right)$

p.9.a. The model fit is given below. Obtain simultaneous 95% confidence Intervals for β_0 and β_1:

```
> rock.mod <- nls(CLI ~ b0*(QC^b1), start=c(b0=1, b1=-0.1))
> summary(rock.mod)

Formula: CLI ~ b0 * (QC^b1)

Parameters:
   Estimate Std. Error t value Pr(>|t|)
b0 16.16447    3.21159   5.033 1.36e-05 ***
b1 -0.25024    0.06667  -3.754 0.000615 ***
---

Residual standard error: 2.449 on 36 degrees of freedom
```

β_0 ________________________ β_1 ________________________

p.9.b. Give the fitted values for X=0%, 40%, and 80% Quartz and identify them on the graph.



$\hat{Y}_0 =$ ________________ $\hat{Y}_{40} =$ ________________ $\hat{Y}_{80} =$ ________________

QC.10. A study considered the relation between Total Weight of Octopus beaks (X, in grams) and the number of increments (used in aging) on the lateral wall (Y) for a sample of n = 30 octopi.

The authors considered the following model: $Y = \beta_1 X^{\beta_2} + \varepsilon \quad \varepsilon \sim N\left(0, \sigma^2\right)$

p.10.a. What is E{Y} when X=0? ________________

p.10.b. Assuming β_1, $\beta_2 > 0$, What is the shape of E{Y} with respect to X when $\beta_2 = 1$? $\beta_2 > 1$? $\beta_2 < 1$? Match each with the description.

Bends up ________________ Bends Down ________________ Straight line with positive slope ____________

p.10.c. The nonlinear regression model was fit giving the following results. Obtain an approximate 95% Confidence Interval for β_2.

```
Formula: latWall ~ b1 * (totWt^b2)

Parameters:
   Estimate Std. Error t value Pr(>|t|)
b1 32.84397    7.50965   4.374 0.000153 ***
b2  0.22841    0.03013   7.580 2.95e-08 ***

Residual standard error: 21.32 on 28 degrees of freedom
```

p.10.d. Give the fitted values for octopus' of Total weights: 1000, 3000, and 5000 grams.

$\hat{Y}_{1000} =$ ______________________ $\hat{Y}_{3000} =$ ______________________ $\hat{Y}_{5000} =$ ______________________

QC.11. An experiment was conducted to fit an exponential decay model, relating wet foam height (Y, in cm) to time since pouring (X, in seconds) for Shiner Bock. The model fit is given below, where measurements were made at n = 13 points.

$$Y = \beta_0 e^{-\beta_1 X} + \varepsilon \qquad \varepsilon \sim N\left(0, \sigma^2\right) \qquad \beta_0, \beta_1 > 0$$

p.11.a. Give the expected value of Y: at the time of pouring (X=0) ____________ as $X \rightarrow \infty$ ________________

p.11.b. Multiplicatively, how much does height change on average as time since pouring increaseses by 1 second?

p.11.c. The nonlinear regression results are given below.

```
Formula: Y ~ b0 * exp(-b1 * X)

Parameters:
    Estimate Std. Error t value Pr(>|t|)
b0 1.639e+01  3.206e-01   51.11 1.98e-14 ***
b1 6.417e-03  2.587e-04   24.80 5.26e-11 ***
```

Obtain 95% Confidence Intervals for β_0, β_1 :

β_0 ____________________________________ β_1 ____________________________________

p.11.d. Give estimates of the foam height at X=0, 150, and 300 seconds

$\hat{Y}_0 =$ ____________________ $\hat{Y}_{150} =$ ____________________ $\hat{Y}_{300} =$ ____________________

p.11.e. At what time is it estimated to decay by ½ from the peak?

QC.12. A nonlinear regression model was fit for steel sheets, relating plain strain form limit (Y) to scaled ultimate tensile stress (X = UTS-200). The relationship is modelled with following equation and was based on an experiment with n = 56 steel sheets (with following expected signs for the the regression coefficients).

Answer all parts in terms of X, not UTS.

$$Y = \beta_0 + \beta_1 e^{-\beta_2 X} + \varepsilon \quad \varepsilon \sim NID\left(0, \sigma^2\right) \quad \beta_0 > 0, \beta_1 > 0, \beta_2 > 0$$

p.12.a. Give the intercept and asymptote (limit as X → ∞) in terms of the model parameters.

Intercept ___________________ Asymptote ______________________

p.12.b. At what point X^* does the model reach half-way between the intercept and asymptote?

p.12.c. The estimated regression coefficients and variance-covariance matrix are given below. Obtain approximate 95% Confidence Intervals for β_0, β_1, $\beta_0+\beta_1$

Parameter	Estimate		V{B}		
B0	0.271242		0.000191	0.001077	0.000032
B1	0.543144		0.001077	0.030845	0.000600
B2	0.012461		0.000032	0.000600	0.000013

Hint: $\hat{V}\left\{\hat{\beta}_0\right\} = 0.000191 \quad \hat{V}\left\{\hat{\beta}_1\right\} = 0.030845 \quad \hat{COV}\left\{\hat{\beta}_0, \hat{\beta}_1\right\} = 0.001077$

β_0 ________________________ β_1 ________________________ $\beta_0 + \beta_1$ ________________________