STA 6166 – Exam 1 – Spring 2016 **PRINT** Name ANSWER KEY 100

Q.1. Chicago food establishments are classified by 3 levels of Risk (High, Medium, and Low). The proportions (probabilities) are: P(High) = .65, P(Medium) = .22, P(Low) = .13. The probabilities of Failing inspection are .21 among High risk, .23 among Medium Risk, and .33 among Low Risk.

p.1.a. Compute the probability a randomly selected establishment Fails inspection.

Type	P(Type)	P(F | Type)	P(Type & Fail)	P(Type | F)
H	.65 1	.21 1	.1365 1	.5935 2
M	.22 1	.23 1	.0506 1	.2200 2
L	.13 1	.33 1	.0429 1	.1865 2
			.2300 1	1.00

10

p.1.b. Compute the probability of each risk type, given the establishment fails inspection.

P(High Risk | Fail) = .5935 (2) **P(Medium Risk | Fail)** = .2200 (2) **P(Low Risk | Fail)** = .1865 (2)

Q.2. The June monthly rainfall totals (in inches) for a sample of 5 Orlando years were: 10, 5, 5, 6, 9.

Give the sample mean, median, and standard deviation of the monthly rainfall totals (show all work).

i	y_i	rank	$(y_i-\bar{y})^2$
1	10	5	9
2	5	1.5	4
3	5	1.5	4
4	6	3	1
5	9	4	4
	35		22

$\bar{y} = 7$ (8)

$s^2 = \frac{22}{5-1} = 5.50$

$s = \sqrt{5.50} = 2.35$

Mean = 7 (7) **Median** = 6 (3) **Std. Deviation** = 2.35 (4)

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Q.3. An examination is given with n = 5 multiple-choice questions, each with 4 choices, and 1 correct answer. A student arrives for the exam completely unprepared, and will randomly guess on each question.

p.3.a. What is the probability the student will get at least 1 correct. $Y \sim Bin(n=5, \pi=.25)$

$$P(Y \geq 1) = 1 - P(0) = 1 - (.75)^5 = 1 - .2373 = .7627$$

8

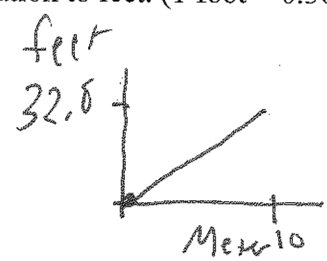
p.3.b. What are the mean and standard deviation of the number correct answers if this exam was given many times to people randomly guessing answers?

4 $E\{Y\} = n\pi = 5(.25) = 1.25$

4 $\sigma_Y = \sqrt{n\pi(1-\pi)} = \sqrt{5(.25)(.75)} = .9682$

Q.4. Elite female hammer thrower Anita Włodarczyk has a competitive mean distance thrown of 73.36 meters and standard deviation of 2.74 meters. Translate her mean and standard deviation to feet. (1 foot = 0.3048 meters).

$\mu_M = 73.36$ $\quad \sigma_M = 2.74$



$1 \text{ meter} = \frac{1}{.3048} \text{ feet} = 3.2808 \text{ Feet}$

$\mu_F = 73.36(3.2808) = 240.68$ 3

$\sigma_F = 2.74(3.2808) = 8.99$ 3

Q.5. An engineer is interested in estimating the population mean lifetime of a new type of light bulb. Based on a pilot study, they estimate the standard deviation to be 100 hours. How large of a sample will be needed to have a margin of error of 10 hours with 95% confidence?

$\sigma \approx 100 \quad z_{.025} = 1.96 \quad E = 10$

$10 = 1.96\left(\frac{100}{\sqrt{n}}\right) \Rightarrow \sqrt{n} = 1.96\left(\frac{100}{10}\right) = 19.6$ 2 2 2

2 $\Rightarrow n = 19.6^2 \approx 385$

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Q.6. A random sample of n = 16 female runners at the Washington, DC marathon had a sample mean speed of 6.04 miles per hour with a sample standard deviation of 0.78 miles per hour.

$n = 16 \quad \bar{y} = 6.04 \quad s = 0.78$

p.6.a. Compute the estimated standard error of the sample mean.

$$S_{\bar{y}} = \frac{s}{\sqrt{n}} = \frac{0.78}{\sqrt{16}} = 0.195$$

4

p.6.b. Give the degrees of freedom corresponding to the answer in p.6.a. $n-1 = 16-1 = 15$

2

p.6.c. Obtain a 95% confidence interval for μ (the population mean of all female runners in that marathon).

3

$$t_{.025,15} = 2.131 \Rightarrow 6.04 \pm 2.131(0.195)$$

$$= 6.05 \pm 0.42 = (5.63, 6.47) \quad 3$$

Q.7. A researcher wishes to test whether the population mean time for her factory's workers to complete a task exceeds 30 minutes. She takes a random sample of 64 workers from her firm's large factory, and measures the time it takes each worker to complete the task.

p.7.a. Give the null and alternative hypotheses $H_0: \mu \leq 30 \quad H_A: \mu > 30$ 3

p.7.b. The sample mean was 36 minutes and the sample standard deviation was 24 minutes. Compute the test statistic.

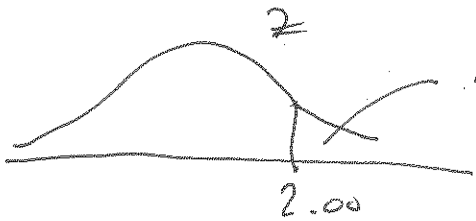
$$\bar{y} = 36 \quad s = 24 \quad n = 64 \Rightarrow \frac{s}{\sqrt{n}} = \frac{24}{\sqrt{64}} = 3$$

2 2

$$T.S. \quad z_{obs} = \frac{36-30}{3} = 2.00$$

3

p.7.c. Obtain the p-value, based on the z-distribution.



.0228 3

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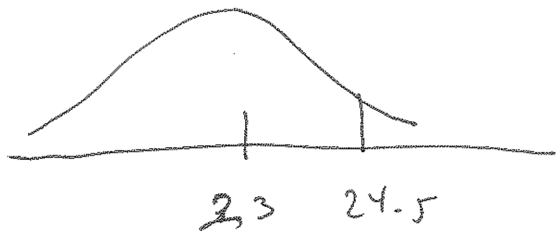
Q.8. Body Mass Indices (BMI) for English Premier League (EPL) football players are approximately normally distributed with a mean of 23.00 and standard deviation of 1.70.

p.8.a. What is the probability a randomly selected EPL player has a BMI above 24.5?



$$z = \frac{24.5-23}{1.70} = \frac{1.50}{1.70} = 0.88$$ 4

2

$$P(Z \geq .88) = .1894$$

p.8.b. Between what 2 BMI levels do the middle 50% of all EPL players fall?

3

$$P(-c \leq Z \leq c) \Rightarrow c = .675$$

$$\mu + .675\sigma = 23 + \underbrace{.675(1.70)}_{1.15}$$

$$23 - .675(1.70)$$

3

$$\Rightarrow (21.85, 24.15)$$

p.8.c. What is the sampling distribution of sample means of sample size = 16 from this population? Give the distribution symbolically and draw a graph of it.

$$\bar{y} \sim N\left(23.00, \sigma_{\bar{y}} = \frac{1.70}{\sqrt{16}} = 0.425\right)$$

3 2 3

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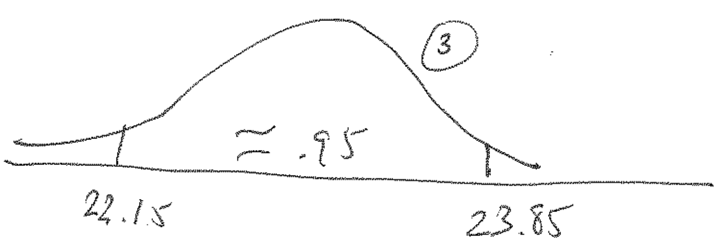

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