TOTAL = 135

STA 6166 – Exam 1 – Fall 2016 **PRINT** Name ANSWER KEY

Q.1. A random sample of n = 5 Kentucky Derby winning times were selected and measured to be: 123, 124, 128, 131, 124 seconds. Compute the sample mean, median, and standard deviation. Show all work.

	y	$(y-\bar{y})^2$
	123	9
	124	4
	128	4
	131	25
	124	4
Sum	630	46
Mean	126	

$s^2 = \frac{46}{5-1} = 11.5$

$s = \sqrt{11.5} = 3.39$

Mean: 5 126 Median: 3 124 Standard Deviation: 6 3.39

Q.2. Which of the following two statements is true? Mark the correct answer.

4

i) Median and Interquartile Range are not affected by outliers, Mean and Standard Deviation are affected by outliers.

ii) Median and Interquartile Range are affected by outliers, Mean and Standard Deviation are not affected by outliers.

Q.3. An equine researcher reports that the running speed of a particular thoroughbred horse has a mean of $\mu = 15$ meters per second and a standard deviation of $\sigma = 0.7$ meters per second.
Convert these to feet per second: 1 meter = 3.281 feet.

feet, meters

$Y = 3.281X$

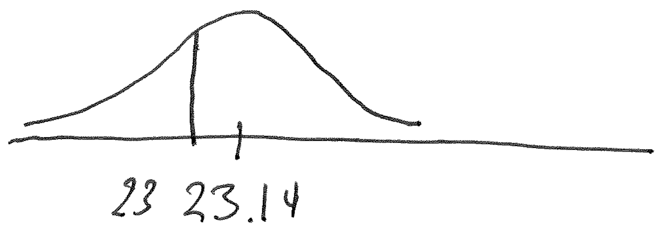
5
p.3.a. Mean (feet/second): $3.281(15) = 49.215$

5
p.3.b. Standard Deviation: (feet/second): $3.281(0.7) = 2.297$

28

Q.4. Among all 2013-2014 Women's National Basketball Association players, Body Mass Indices (BMI) are approximately normally distributed with $\mu_{WNBA} = 23.14$ and $\sigma_{WNBA} = 2.11$.

p.4.a. What is the probability that a randomly selected player has a BMI of at least 23?

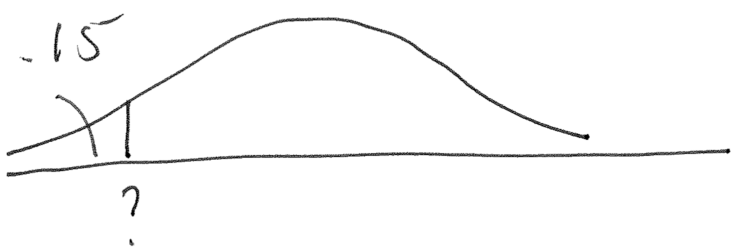


$P(Y \geq 23) = P\left(Z \geq \frac{23-23.14}{2.11} = -0.07\right)$

$= 1 - P(Z \leq -.07) = 1 - .4721$

$= \underline{.5279}$ 8

p.4.b. Below what BMI level do the lowest 15% of players fall?



$Z_{.15} \doteq -1.04$

$23.14 - 1.04(2.11)$

$= 23.14 - 2.19$

$= \underline{20.95}$

8

p.4.c. If a random sample of n = 25 players, were selected, between what 2 BMI values (BMI_{Low}, BMI_{High}) would the sample mean fall between with probability .95?

$\frac{\sigma}{\sqrt{n}} = \frac{2.11}{\sqrt{25}} = \frac{2.11}{5} = 0.422$ 6

4 $1.96(.422) = 0.83$ $\quad$ $23.14 \pm 0.83 = (22.31, 23.97)$

$2.0\cancel{6}(.422) = 0.84$ $\quad$ $(22.30, 23.98)$

2

BMI_{Low} = 22.31 $\qquad$ BMI_{High} = 23.98

26

Q.5. A manufacturing company has 3 plants: A, B, and C. Plant A produces 45% (0.45) of their product, B produces 35% (0.35), and C produces 20% (0.20). Among items produced by Plant A, 10% (0.10) are considered to be High Quality, for Plant B, 20% (0.20) are High Quality, and for Plant C, 40% (0.40) are High Quality.

p.5.a. What proportion of the company's items are High Quality (HQ)? $P(HQ) = .195$ 12

Plant	P(Plant)	P(HQ | P)	P(P & HQ)	P(P | HQ)
A	.45	.10	.045	.045/.195 = .231
B	.35	.20	.070	.070/.195 = .359
C	.20	.40	.080	.080/.195 = .410
			.195	1.000

p.5.b. An item is selected at random, and is identified as High Quality (HQ). What is the Probability it was produced at Plant A? At Plant B? At plant C?

4 each

P(Plant A | HQ) = .231 P(Plant B | HQ) = .359 P(Plant C | HQ) = .410

Q.6. For 50' rolls of fabric, the number of imperfections follows a Poisson distribution with a mean of 3.6.

p.6.a. What is the probability a randomly selected roll has either 0 or 1 imperfections.

$P(0) = e^{-3.6} = .0273$ 4

$P(1) = 3.6e^{-3.6} = .0984$ 4

$P(Y \leq 1) = .1257$ 2

p.6.b. What would be the approximate sampling distribution of means of samples of n = 81 rolls. Give the mean, standard error, and approximate shape (distribution).

$\sigma_{\bar{y}} = \sqrt{\frac{\sigma^2}{n}} = \sqrt{\frac{3.6}{81}} = 0.2108$ 8

3

Mean: 3.6 Standard Error: .2108 Approximate Shape: 3 Normal

48

Q.7. A diagnostic test has a sensitivity of .90. That is, among people with the condition of interest, 90% will correctly test positive. A sample of 15 people who have the condition will be given the diagnostic test.

p.7.a. What is the expected number of the 15 who will test positive? What is the standard deviation of the number who will test positive?

$$E\{Y\} = n\pi = 15(.9) = 13.5 \quad 5$$

$$\sigma_Y = \sqrt{n\pi(1-\pi)} = \sqrt{15(.9)(.1)} = \sqrt{1.35} \approx 1.16 \quad 5$$

Mean μ: 13.5 Standard Deviation σ: 1.16

p.7.b. What is the probability either 14 or 15 will test positive?

$$P(14) = \frac{15!}{14!1!}(.9)^{14}(.1)^{15-14} = 15(.9)^{14}(.1) = .3432 \quad 4$$

$$P(15) = \frac{15!}{15!0!}(.9)^{15}(.1)^{15-15} = .9^{15} = .2059 \quad 4$$

$$\Rightarrow P(Y \geq 14) = .3432 + .2059 = .5491 \quad 2$$

Q.8. On an isolated island, there is a closed population of birds. The birds are classified by Gender (Male / Female) and Color (Red / Green). There are no mixed gender or mixed color birds (in terms of appearance). You are given the following contingency table. Let **F** be the event that a randomly selected bird is Female, and **G** be the event that the bird is Green.

Gender\Color	Red	Green	Total
Male	800	200	1000
Female	1200	300	1500
Total	2000	500	2500

p.8.a. P(**F**) = $\frac{1500}{2500} = 0.60$ 5

p.8.b. P(**F**|**G**) = $\frac{300}{500} = 0.60$ 5

p.8.c. Are **F** and **G** independent events? Yes No 3

33

STA 6166 – Exam 1 – Fall 2016 **PRINT** Name Answer Key 135

Q.1. A random sample of n = 5 Kentucky Derby winning times were selected and measured to be: 124, 125, 129, 132, 125 seconds. Compute the sample mean, median, and standard deviation. Show all work.

Rank	y	$(y-\bar{y})^2$
1	124	9
2.5	125	4
4	129	4
5	132	25
2.5	125	4
	635	46

$s^2 = \frac{46}{4} = 11.5$ $\quad s = \sqrt{11.5} = 3.39$

$\bar{y} = 127$

Mean: 5 127 Median: 3 125 Standard Deviation: 6 3.39

Q.2. Which of the following two statements is true? Mark the correct answer.

4 i) Median and Interquartile Range are affected by outliers, Mean and Standard Deviation are not affected by outliers.

ii) Median and Interquartile Range are not affected by outliers, Mean and Standard Deviation are affected by outliers.

Q.3. An equine researcher reports that the running speed of a particular thoroughbred horse has a mean of $\mu = 16$ meters per second and a standard deviation of $\sigma = 0.5$ meters per second.
Convert these to feet per second: 1 meter = 3.281 feet.

5

p.3.a. Mean (feet/second): $3.281(16) = 52.496$

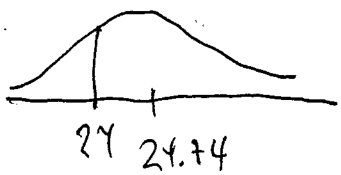
5

p.3.b. Standard Deviation: (feet/second): $3.281(.5) = 1.6405$

28

Q.4. Among all 2013-2014 National Basketball Association players, Body Mass Indices (BMI) are approximately normally distributed with $\mu_{NBA} = 24.74$ and $\sigma_{NBA} = 1.72$.

p.4.a. What is the probability that a randomly selected player has a BMI of at least 24?



$$z_{24} = \frac{24-24.74}{1.72} = \frac{-.74}{1.72} = -0.43$$

$$P(z \ge -.43) = 1 - P(z \le -.43) = 1 - .3336$$

$$= .6664$$ 8

p.4.b. Below what BMI level do the lowest 20% of players fall?

$$q_{.20} = 24.74 + z_{.20}(1.72)$$

$$= 24.74 - 0.84(1.72)$$

$$= 24.74 - 1.44 = 23.30$$ 8

p.4.c. If a random sample of n = 16 players, were selected, between what 2 BMI values (BMI_{Low} , BMI_{High}) would the sample mean fall between with probability .95?

$$\sigma_{\bar{Y}} = \frac{1.72}{\sqrt{16}} = 0.43$$ 6

4

$$24.74 \pm \underbrace{1.96(.43)}_{.8428} = (23.8972, 25.5828)$$

$$24.74 \pm \underbrace{2(.43)}_{.86} = (23.88, 25.60)$$

2

BMI_{Low} = 23.88 BMI_{High} = 25.60

26

Q.5. A manufacturing company has 3 plants: A, B, and C. Plant A produces 40% (0.40) of their product, B produces 35% (0.35), and C produces 25% (0.25). Among items produced by Plant A, 20% (0.20) are considered to be High Quality, for Plant B, 30% (0.30) are High Quality, and for Plant C, 50% (0.50) are High Quality.

p.5.a. What proportion of the company's items are High Quality (HQ)?

Plant	P(P)	P(HQ|P)	P(P & HQ)	P(P|HQ)
A	.40	.20	.080	.080/.310 = .258
B	.35	.30	.105	.105/.310 = .339
C	.25	.50	.125	.125/.310 = .403
	1.00		.310 12	1

p.5.b. An item is selected at random, and is identified as High Quality (HQ). What is the Probability it was produced at Plant A? At Plant B? At plant C?

1 each

P(Plant A | HQ) = .258 P(Plant B | HQ) = .339 P(Plant C | HQ) = .403

Q.6. For 50' rolls of fabric, the number of imperfections follows a Poisson distribution with a mean of 2.4.

p.6.a. What is the probability a randomly selected roll has either 0 or 1 imperfections.

$P(0) = e^{-2.4} = .0907$ (4)

$P(1) = 2.4e^{-2.4} = .2177$ (4)

$P(Y \le 1) = .3084$ (2)

p.6.b. What would be the approximate sampling distribution of means of samples of n = 64 rolls. Give the mean, standard error, and approximate shape (distribution).

$\bar{Y} \sim N\left(2.4, \sqrt{\frac{2.4}{64}} = \sqrt{.0375} = .1936\right)$ (8)

(3) (3)

Mean: 2.4 Standard Error: .1936 Approximate Shape: Normal

-15

Q.7. A diagnostic test has a sensitivity of .95. That is, among people with the condition of interest, 95% will correctly test positive. A sample of 10 people who have the condition will be given the diagnostic test.

p.7.a. What is the expected number of the 10 who will test positive? What is the standard deviation of the number who will test positive?

$$Y \sim Bin(n=10, \pi = .95)$$

$$E(Y) = 10(.95) = 9.5 \quad 5$$

$$\sigma_Y = \sqrt{10(.95)(1-.95)} = \sqrt{.475} = .689 \quad 5$$

Mean μ: 9.5 Standard Deviation σ: .689

p.7.b. What is the probability either 9 or 10 will test positive?

$$P(9) = \frac{10!}{9!1!} .95^9(.05^1) = 10(.95)^9(.05) = .3151 \quad 4$$

$$P(10) = .95^{10} = .5987 \quad 4$$

$$P(Y \geq 9) = .3151 + .5987 = .9138 \quad 2$$

Q.8. On an isolated island, there is a closed population of birds. The birds are classified by Gender (Male / Female) and Color (Red / Green). There are no mixed gender or mixed color birds (in terms of appearance). You are given the following contingency table. Let **M** be the event that a randomly selected bird is Male, and **R** be the event that the bird is Red.

Gender\Color	Red	Green	Total
Male	800	200	1000
Female	600	150	750
Total	1400	350	1750

p.8.a. P(**M**) = $\frac{1000}{1750} = .5714$ 5

p.8.b. P(**M**|**R**) = $\frac{800}{1400} = .5714$ 5

3

p.8.c. Are **M** and **R** independent events? **Yes** **No**

33