# Probability and Probability Distribution Problems Solutions

**Q.1.** $P(G|M) = .20 \quad P(G|F) = .10 \quad P(M) = .40 \quad P(F) = .60$

**P.1.a** $P(G) = P(G \cap M) + P(G \cap F) = P(M)P(G|M) + P(F)P(G|F)$

$= .40(.20) + .60(.10) = .08 + .06 = .14$

**P.1.b.** $P(M|G) = \dfrac{P(M \cap G)}{P(G)} = \dfrac{.08}{.14} = \dfrac{4}{7} = .5714$

**Q.2.** $Y \equiv$ High temperature on a randomly selected day in °C

$\mu_Y = 20 \qquad X \equiv$ High temp in °F $= 32 + \frac{9}{5}Y$

$\Rightarrow \mu_X = 32 + \frac{9}{5}\mu_Y = 32 + \frac{9}{5}(20) = 32 + 9(4) = 68$

**Q.3.** $Y \equiv$ # Light Bulbs w/ Lifetime > 800 hours in 10 sampled

$Y \sim Bin(n = 10, \pi = 0.80)$

$P(Y = 10) = \binom{10}{10}(.80)^{10}(1 - .80)^{0} = .8^{10} = .1074$

**Q.4.** Scores $\equiv Y \sim N(\mu = 550, \sigma = 100)$

$P(Y > 700) = P\left(\frac{Y - \mu}{\sigma} > \frac{700 - 550}{100} = 1.50\right)$

$= 1 - P(Y \leq 1.50) = 1 - .9332 = .0668$

**Q.5.** $Y \sim Bin(n=5, \pi=.05)$

$$P(Y=0) = \binom{5}{0}(.05)^0(.95)^{5-0} = .95^5 = .7738$$

**Q.6.** $P(C|M) = \frac{40}{25636} = .00156$ $P(C|M) = \frac{60}{27167} = .00221$

**Q.7.** FALSE ($\hat{\pi} = .85$, not $\pi = .85$)

**Q.8.** $Y \sim Bin(n=6, \pi=.15)$ $P(Y=0) = \binom{6}{0}(.15)^0(.85)^{6-0} = .3771$

**Q.9.** $F^+ \equiv$ Flu-Positive $T^+ \equiv$ Test positive

$$P(T^+|F^+) = \frac{664}{748} = .8877 \qquad P(T^-|F^-) = \frac{80}{82} = .9756$$

**Q.10.** $Y \sim Bin(n=500, \pi=.01)$

$$E\{Y\} = n\pi = 500(.01) = 5$$

$$P(Y \le 1) = P(0) + P(1) = \binom{500}{0}(.01)^0(.99)^{500} + \binom{500}{1}(.01)^1(.99)^{499}$$

$$= \frac{500!}{0!\,500!}(.01)^0(.99)^{500} + \frac{500!}{1!\,499!}(.01)^1(.99)^{499}$$

$$= (.99)^{500} + 500(.01)(.99)^{499} = .00657 + .03184 = .03975$$

**Q.11.** $\text{Prob}\{\text{Same Review}\} = \frac{400+300}{1000} = 0.70$

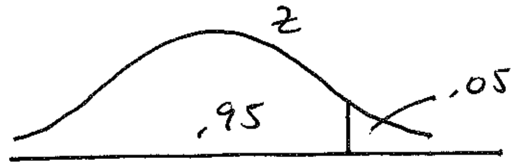
$$\text{Prob}\{R_2^+ | R_1^-\} = \frac{100}{400} = 0.25$$

**Q. 12.** $Y_m \sim N(167, 6) \qquad Y_F \sim N(160, 5)$

**P.12.a.** $P(Y_m \geq 175) = P\left(Z_m = \frac{Y_m - \mu_m}{\sigma_m} \geq \frac{175-167}{6} = 1.33\right)$

$= 1 - P(Z \leq 1.33) = 1 - .9082 = .0918$

**P.12.b.**




$Z_{.95} = 1.645$

$\Rightarrow$ $95^{th}$ %-ile for Females: $\mu_F + 1.645\,\sigma_F = 160 + 1.645(5)$
$= 168.225$

**Q. 13.** $P(F) = .60 \quad P(M) = .40 \quad P(C|F) = .20 \quad P(C|M) = .40$

**P.13.a.** $P(C) = P(F)\,P(C|F) + P(M)\,P(C|M) = .6(.2) + .4(.4) = .12 + .16 = .28$

**P.13.b.** $P(M|C) = \frac{P(MC)}{P(C)} = \frac{.16}{.28} = \frac{4}{7} = .5714$

**Q.14.** $Y \equiv$ # correct responses on test $\quad Y \sim Bin(n=6, \pi = .20)$

**P.14.a.** $E\{Y\} = n\pi = 6(.20) = 1.20$

**P.14.b.** $P(Y=0) = \binom{6}{0}(.2)^0(.8)^{6-0} = (.8)^6 = .2621$

**Q.15.** $Y \sim N(2100, 500) \qquad n = 25 \Rightarrow \bar{Y} \sim N\left(2100, \frac{500}{\sqrt{25}} = 100\right)$

$P(\Sigma Y_i \geq 80000) = P\left(\bar{Y} \geq \frac{80000}{25} = 3200\right) = P\left(Z \geq \frac{3200-2100}{100} = 11\right)$

$\approx 0$

Q.16. $D^+ \equiv$ Disease Positive $\quad T^+ \equiv$ Test Positive

Sensitivity $= .95 = P(T^+ | D^+)$ $\quad$ Specificity $= P(T^- | D^-) = .95$

Prevalence $= P(D^+) = .01$

P.16.a. $P(T^+) = P(T^+ \cap D^+) + P(T^+ \cap D^-)$

$$= P(D^+)P(T^+|D^+) + P(D^-)P(T^+|D^-)$$

$$= .01(.95) + .99(1 - .95) = .059$$

P.16.b. $P(D^+|T^+) = \dfrac{P(T^+ \cap D^+)}{P(T^+)} = \dfrac{.01(.95)}{.059} = .1610$

Q.17. P.17.a) P(Student Smokes ; @ least 1 Parent smokes)

$$= .07 + .08 = .15$$

P.17.b. P(Neither Parent Smokes | Student Smokes) $= \dfrac{.03}{.18} = .1667$

Q.18. $Y \sim N(\mu = 454, \sigma = 8)$ $\quad n = 25$

$$\Rightarrow \bar{Y} \sim N\left(\mu_{\bar{Y}} = 454, \sigma_{\bar{Y}} = \frac{8}{\sqrt{25}} = \frac{8}{5} = 1.6\right)$$

Q.19. School System salaries in (18,000 - 40,000)

P.19.a. Every teacher gets \$1000 raise $\Rightarrow$ Mean increases \$1000
SD remains same

P.19.b. Every teacher gets 5% raise $\Rightarrow$ raise ranges from \$900 - \$2000
Mean increases by 5% $\quad$ SD increases by 5%

Q.20.

| Color | $P(C)$ | $P(T\|C)$ | $P(T \cap C)$ | $P(C\|T)$ |
|---|---|---|---|---|
| Red | .40 | .05 | .020 | .1739 |
| Yellow | .30 | .10 | .030 | .2609 |
| Black | .20 | .20 | .040 | .3478 |
| Green | .10 | .25 | .025 | .2174 |
| Total | 1.00 | — | .115 | 1.000 |

Q.21. $Y \equiv$ # successful Penalty Kicks

$Y \sim Bin(n=3, \pi=0.80)$

P.21.a. $P(Y=3) = \binom{3}{3}(.80)^3(.20)^{3-3} = .8^3 = .512$

P.21.b. $P(Y=0) = \binom{3}{0}(.80)^0(.20)^{3-0} = .2^3 = .008$

P.21.c. $\hat{\pi} \dot{\sim} Normal\left(\mu_{\hat{\pi}} = \pi = .80,\ \sigma_{\hat{\pi}} = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{.8(.2)}{64}} = .05\right)$

Q.22. 3 phone brands $P(LG) = .25$ $P(M) = .40$ $P(N) = .35$

$LS \equiv$ Low SAR Level $P(LS|LG) = .24$ $P(LS|M) = .20$ $P(LS|N) = .28$

P.22.a. $P(LS) = .25(.24) + .40(.20) + .35(.28)$

$= .060 + .080 + .098 = .238$

P.22.b.) $P(LG|LS) = \frac{.060}{.238} = .2521$

$P(M|LS) = \frac{.080}{.238} = .3361$

$P(N|LS) = \frac{.098}{.238} = .4118$

**Q.23.** $Y \sim Bin(n=10, \pi = 0.4)$

**P.23.a.** $P(Y=4) = \binom{10}{4}(.4)^4(.6)^6 = \frac{10!}{4!\,6!}(.4)^4(.6)^6 =$

$= 210(.0256)(.046656) = .2508$

**P.23.b.** $E\{Y\} = n\pi = 10(0.4) = 4.0$

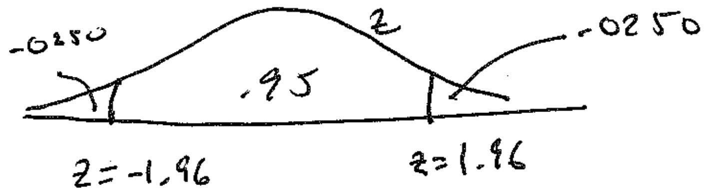
$V\{Y\} = n\pi(1-\pi) = 10(.4)(.6) = 2.4$

**Q.24.** $Y \sim N(26.50, 1.45)$

**P.24.a.** $P(Y \leq 25.0) = P\left(Z = \frac{Y-\mu}{\sigma} \leq \frac{25.0-26.50}{1.45} = -1.03\right)$

$= P(Z \leq -1.03) = .1515$

**P.24.b.**

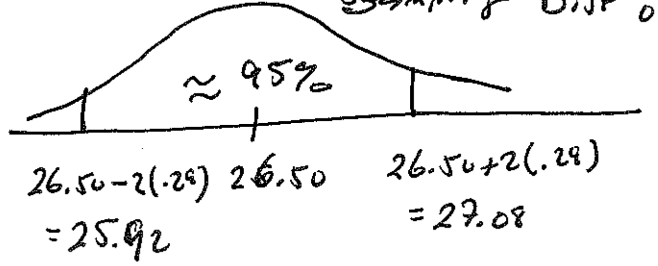



$\mu \pm 1.96\sigma \equiv 26.50 \pm \underbrace{1.96(1.45)}_{2.842} = (23.658, 29.342)$

$\Rightarrow P(23.658 \leq Y \leq 29.342) = .95$

**P.24.c.** $n=25 \Rightarrow \bar{Y} \sim N\left(\mu_{\bar{Y}} = 26.50,\ \sigma_{\bar{Y}} = \frac{1.45}{\sqrt{25}} = 0.29\right)$

~~S~~ Sampling Dist of $\bar{Y}$



Q.25. $E\{Y_F\} = \mu_F = 91.4°F$ $\sigma_F = 8.3°F$

$$Y_C = 0.56 Y_F - 17.78$$

$$\mu_C = 0.56\,\mu_F - 17.78 = 0.56(91.4) - 17.78 = 33.404$$

$$\sigma_C = |0.56|\sigma_F = 0.56(8.3) = 4.648$$