<u>Inference Concerning Mean and Median Values</u>

Q.1. In tests of hypotheses what is a Type I error?

a) rejecting Ho when it is true
b) failing to reject Ho when it is false

Q.2. A measure of how unusual the observed outcome is, if Ho were true, is called

a) the p-value
b) the test statistic
c) the rejection region

Q.3. Consider a test of Ho: mean $\mu = 50$ versus Ha: $\mu > 50$ using a sample of $n = 25$ and a significance level of 5%. Is the power of the test increased, decreased, or unchanged if

p.3.a. the sample size is increased to 50? **increased**

p.3.b. the significance level is changed to 10%? **increased**

p.3.c. the true value of the mean is 65 rather than 75? **decreased**

Q.4. For each of the following situations, is the p-value above or below 0.05 for the value of the test statistic (TS) shown?

p.4.a. Ho: $\mu = 35$, Ha: $\mu > 35$, $n = 26$, TS = 2.781 > 0.05 or **< 0.05**

p.4.b. Ho: $\mu = 45$, Ha: $\mu \neq 45$, $n = 12$, TS = -1.749 **> 0.05** or < 0.05

Q.5. The significance level for a hypothesis test is

i. the probability of not rejecting Ho when Ho is false.
ii. the probability of rejecting Ho when Ho is false.
iii. the probability of not rejecting Ho when Ho is true.
iv. the probability of rejecting Ho when Ho is true.

Q.6. For a particular sample, the 95% confidence interval for the population mean is [23, 37]. Which of the following statements is true?

i. 95% of the population is in the interval [23, 37].
ii. 95% of the samples from this population have means in the interval [23, 37].
iii. This interval was calculated using a procedure that gives correct intervals 95% of the time.

Q.7. In conducting a hypothesis test regarding a mean, the larger the P-value, the stronger the evidence against the null hypothesis. True or **False**

Q.8. All else being equal, as we increase the sample size, we increase the width of a Confidence Interval for μ **FALSE**

Q.9. A method of spraying for rust mites has been conducted on a random sample of 100 10-acre plots in citrus groves in Florida. The sample mean and standard deviation for the yield of fruit is 750 and 200, respectively. Construct a 95% confidence interval for μ, the population mean yield per 10-acre plot.

$$n=100 \quad \overline{y}=750 \quad s=200 \quad z_{.025}=1.96 \quad t_{.025,99}=1.984 \quad \hat{SE}\left\{\overline{Y}\right\}=\frac{s}{\sqrt{n}}=\frac{200}{\sqrt{100}}=20$$

$$\text{"z"-interval:} \quad 750 \pm 1.96(20) \equiv 750 \pm 39.2 \equiv (710.8, 789.2)$$

$$\text{"t"-interval:} \quad 750 \pm 1.984(20) \equiv 750 \pm 39.68 \equiv (710.32, 789.68)$$

Q.10. A researcher wishes to estimate the population mean amount of bacteria per plate of culture when a fixed amount of antibiotic is applied. Previous work has shown that the standard deviation of the amount of bacteria present is approximately 18 cm^2. Use this to determine the number of cultures to be run to estimate the true mean within 2 cm^2 with 95% confidence. (That is, the half-with of a 95% confidence interval will be 2).

$$\sigma \approx 18 \quad \alpha=.05 \quad z_{\alpha/2}=z_{.025}=1.96 \quad E=2$$

$$E=z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \quad \Rightarrow \quad n=\frac{z_{\alpha/2}^2\sigma^2}{E^2}=\frac{\left(1.96^2\right)\left(18^2\right)}{2^2}=311.17 \approx 312$$

Q.11. An airplane manufacturer will only purchase aluminum plates of a given size from a supplier if the supplier can demonstrate that the true average pressure needed to crimp the plates exceed 2000 psi. They set up a test by randomly selecting n=64 plates from the supplier and measuring the pressure needed to crimp the plates. Let μ represent the true mean crimping strength. Give the null and alternative hypotheses (note that the airplane manufacturer is putting the burden on the supplier to demonstrate that they meet the specification).

p.11.a. Null Hypothesis H_0: ____________ Alternative H_A: ________________

p.11.b. They observe sample mean=2050 and standard deviation=160. Compute the test statistic.

p.11.c. If they conduct this test at the α=0.05 significance level (Probability of Type I Error), they will conclude that the true mean meets the specification if the test statistic falls in what range of values ___________

p.11.d. Compute the P-value for this test

p.11.e. Has the supplier demonstrated at this level of significance that they meet the specification? **Yes** or **No**

p.11.a. $H_0: \mu \le 2000 \quad H_A: \mu > 2000$

p.11.b. TS: $t_{obs}=\frac{2050-2000}{160/\sqrt{64}}=\frac{50}{20}=2.50$

p.11.c. RR: $t_{obs} \ge t_{.05,63}=1.669$ or $t_{obs} \ge z_{.05}=1.645$

p.11.d. P-value: $P=P\left(t_{63} \ge 2.50\right) \approx P\left(Z \ge 2.50\right)=.0062$

p.11.e. Yes, evidence that true mean exceeds 2000

Q.12. A scientist wishes to estimate the mean breaking strength of a certain type of steel rod within 1.0 psi with 95% confidence. Based on experience with a similar type of steel, she believes the standard deviation is approximately 6.0 psi in individual breaking strengths. How many steel rods should she test?

$n = ((1.96)^2(6.0)^2) / (1.0)^2 = 138.3 \approx 139$

Q.13. A scientist wishes to estimate the mean breaking strength of a certain type of steel rod within 3.0 psi with 95% confidence. Based on experience with a similar type of steel, she believes the standard deviation is approximately 15.0 psi in individual breaking strengths. How many steel rods should she test?

$n \approx 96$

Q.14. A chemist is interested in the mean and variance of batches of a compound received from a supplier. A random sample of n=18 batches are obtained and the amount of the compound is measured from each batch. The sample mean and standard deviation are 23.8 ounces and 0.8 ounces, respectively. Give a 95% confidence interval for the mean amount among all batches from this supplier.

$$n = 18 \quad \overline{y} = 23.8 \quad s = 0.8 \quad \hat{SE}\left\{\overline{Y}\right\} = \frac{s}{\sqrt{n}} = \frac{0.8}{\sqrt{18}} = 0.19 \quad t_{.025,18-1} = 2.110$$

$$95\% \text{ CI for } \mu: \quad 23.8 \pm 2.110(0.19) \equiv 23.8 \pm 0.40 \equiv (23.4, 24.2)$$

Q.15. An author wishes to demonstrate that the average number of typographical errors per page in his (very long) book is less than 1 per page. He wishes to test H_0: $\mu = 1$ H_A: $\mu < 1$ at $\alpha = 0.05$ significance level. Assuming the standard deviation is 0.9, how many pages will need to be sampled to have power = $1-\beta = 0.80$ when in fact the true mean is $\mu = 0.7$

$$H_0: \mu \geq 1 \quad H_A: \mu < 1 \quad \text{TS: } z_{obs} = \frac{\overline{Y}-1}{0.9/\sqrt{n}} \quad \text{RR: } z_{obs} \leq -z_{.05} = -1.645 \quad \Rightarrow \quad \overline{Y} \leq 1 - 1.645\left(\frac{0.9}{\sqrt{n}}\right) = CV_0$$

$$\text{For } \mu_A = 0.7 \text{ want power } = 1-\beta = P(\text{Reject } H_0 \mid \mu = \mu_A = 0.7) = 0.8 = P\left(\overline{Y} \leq CV_0 \mid \overline{Y} \sim N\left(0.7, \frac{0.9}{\sqrt{n}}\right)\right)$$

This means that CV_0 must lie $z_{.20} = 0.84$ standard errors above $\mu_A = 0.7$

$$\Rightarrow \quad CV_0 = 0.7 + 0.84\left(\frac{0.9}{\sqrt{n}}\right) = 1 - 1.645\left(\frac{0.9}{\sqrt{n}}\right) \quad \Rightarrow \quad (0.84 + 1.645)\left(\frac{0.9}{\sqrt{n}}\right) = 1 - 0.7$$

$$\Rightarrow \quad \left(\frac{2.237}{\sqrt{n}}\right) = 0.3 \quad \Rightarrow \quad \sqrt{n} = \frac{2.237}{0.3} = 7.46 \quad \Rightarrow \quad n = 55.6 \approx 56$$

General formula based on large-sample 1-sided z-test (see course notes and slides for non-central t-distribution):

$$n = \frac{\left(z_\alpha + z_\beta\right)^2 \sigma^2}{\left(\mu_0 - \mu_A\right)^2}$$

Q.16. A scientist wishes to estimate the mean length of wood boards with advertised length of 72” within 0.2” with 95% confidence. Based on a small pilot study, she believes the standard deviation is approximately 2” in individual boards. How many boards should she measure? **$n = 384.16 \approx 385$**

Q.17. A random sample of n = 36 cars driving on a residential street had a sample mean of 38 miles/hour, and a standard deviation of 12 miles per hour.

p.17.a. Obtain a 95% Confidence Interval for the true population mean speed of all drivers on the street.

p.17.b. Test whether the population mean speed exceeds the posted speed limit of 35 mph. H_0: $\mu \leq 35$ H_A: $\mu > 35$

Test Statistic: _____________ Rejection Region (Based on Z-dist) ___________ Approximate P-value: ________

$$n = 36 \quad \bar{y} = 38 \quad s = 12 \quad \hat{SE}\{\bar{Y}\} = \frac{12}{\sqrt{36}} = 2 \quad z_{.025} = 1.96 \quad t_{.025,36-1} = 2.030$$

p.17.a. z-interval: $38 \pm 1.96(2) \equiv (34.16, 41.84)$ t-interval: $38 \pm 2.030(2) \equiv (33.94, 42.06)$

p.17.b. TS: $z_{obs} = \frac{38-35}{2} = 1.50$ RR: $z_{obs} \geq z_{.05} = 1.645$ $P = P(Z \geq 1.50) = .0668$

Q.18. A researcher wishes to estimate the true mean mercury level among adult fish in a large lake within 0.2 with 95% confidence. Based on a pilot study, she believes $\sigma \approx 5.0$. How many fish will she need to sample?

n = 2401

Q.19. A researcher is interested in estimating the population mean length of adult female sharks of a particular species within 10 cm with 95% confidence. Based on published research on similar type sharks, she believes that the standard deviation is approximately 70 cm. How many sharks will she need to sample to have her margin of error = 10?

n = 188.2 ≈ 189

Q.20. A random sample of n = 9 women professional basketball players measured for Body Mass Index, resulted in a sample mean of 23.9 and sample standard deviation of 2.7. Complete the following parts, where μ is the population mean among all women professional basketball players:

p.20.a. Compute the estimated standard error of the sample mean. **$2.7 / 9^{1/2} = 0.9$**

p.20.b. Give the degrees of freedom corresponding to the answer in p.20.a. **9-1 = 8**

p.20.c. Obtain a 95% confidence interval for μ **$t_{.025,8} = 2.306$ $23.9 \pm 2.306(0.9) \equiv 23.9 \pm 2.08 \equiv (21.82, 25.98)$**

p.20.d. Test H_0: $\mu = 25$ versus H_A: $\mu \neq 25$ at $\alpha = 0.05$ significance level:

p.20.d.i. Test Statistic **$t_{obs} = (23.9 - 25) / 0.9 = -1.222$**

p.20.d.ii. Rejection Region: **$|t_{obs}| \geq t_{.025,8} = 2.306$**

p.20.d.iii. P-value **> 0.05** or < 0.05

Q.21. A researcher wishes to estimate the population mean amount of bacteria per plate of culture when a fixed amount of antibiotic is applied. Previous work has shown that the standard deviation of the amount of bacteria present is approximately 18 cm^2. Use this to determine the number of cultures to be run to estimate the true mean within 2 cm^2 with 95% confidence. (That is, the half-with of a 95% confidence interval will be 2). **n = 311.2 ≈ 312**

Q.22. An engineer is interested in estimating the population mean lifetime of a new type of light bulb. Based on a pilot study, they estimate the standard deviation to be 100 hours. How large of a sample will be needed to have a margin of error of 10 hours with 95% confidence? **n= 384.16 $\approx$ 385**

Q.23. A random sample of n = 16 female runners at the Washington, DC marathon had a sample mean speed of 6.04 miles per hour with a sample standard deviation of 0.78 miles per hour.

p.23.a. Compute the estimated standard error of the sample mean. **$0.78 / (16)^{1/2} = 0.195$**

p.23.b. Give the degrees of freedom corresponding to the answer in p.6.a. **16-1 = 15**

p.23.c. Obtain a 95% confidence interval for μ (the population mean of all female runners in that marathon).

$t_{.025,15} = 2.131$ $\quad$ $6.04 \pm 2.131(0.195) \equiv 6.04 \pm 0.42 \equiv (5.62, 6.46)$

Q.24. A researcher wishes to test whether the population mean time for her factory's workers to complete a task exceeds 30 minutes. She takes a random sample of 64 workers from her firm's large factory, and measures the time it takes each worker to complete the task.

p.24.a. Give the null and alternative hypotheses **H_0: $\mu \leq 30$ H_A: $\mu > 30$**

p.24.b. The sample mean was 36 minutes and the sample standard deviation was 24 minutes. Compute the test statistic.

$s/n^{1/2} = 24 / (64)^{1/2} = 3$ $\quad$ $t_{obs} = (36 - 30) / 3 = 2.00$

p.24.c. Obtain the p-value, based on the z-distribution.

$P(Z \geq 2.00) = .0228$

Q.25. Body Mass Indices (BMI) for English Premier League (EPL) football players are approximately normally distributed with a mean of 23.00 and standard deviation of 1.70.

p.25.a. What is the probability a randomly selected EPL player has a BMI above 24.5? **.1894**

p.25.b. Between what 2 BMI levels do the middle 50% of all EPL players fall?

$23.00 \pm .6745(1.70) \equiv 23.00 \pm 1.15 \equiv (21.85, 24.15)$

p.25.c. What is the sampling distribution of sample means of sample size = 16 from this population? Give the distribution symbolically and draw a graph of it. **$N(23.00, 1.70 / (16)^{1/2} = 0.425)$ $\quad$ $P(22.15 \leq \text{Ybar} \leq 23.85) \approx 0.95$**