1-Way ANOVA Problems

Q.1. A study is conducted to compare golf ball distances among 4 brands of golf balls. A mechanical driver is set up and hits 12 balls of each brand, and the distance travelled (meters) is measured. The F-test determines that there are differences among the brands. A follow-up comparison based on Tukey's method yields the following table:

Multiple Comparisons

Dependent Variable: y
Tukey HSD

(I) group	(J) group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1	2	-21.40796*	2.51529	.000	-28.1238	-14.6921
	3	-.00121	2.51529	1.000	-6.7171	6.7146
	4	-12.43549*	2.51529	.000	-19.1513	-5.7196
2	1	21.40796*	2.51529	.000	14.6921	28.1238
	3	21.40675*	2.51529	.000	14.6909	28.1226
	4	8.97247*	2.51529	.005	2.2566	15.6883
3	1	.00121	2.51529	1.000	-6.7146	6.7171
	2	-21.40675*	2.51529	.000	-28.1226	-14.6909
	4	-12.43428*	2.51529	.000	-19.1501	-5.7184
4	1	12.43549*	2.51529	.000	5.7196	19.1513
	2	-8.97247*	2.51529	.005	-15.6883	-2.2566
	3	12.43428*	2.51529	.000	5.7184	19.1501

*. The mean difference is significant at the .05 level.

Clearly state what can be said of all pairs of brands at the 0.05 experimentwise error rate.

Brands:

1 vs 2: $\mu_1 < \mu_2$ 1 vs 3: Not significantly different (NSD) 1 vs 4: $\mu_1 < \mu_4$

2 vs 3: $\mu_2 > \mu_3$ 2 vs 4: $\mu_2 > \mu_4$ 3 vs 4: $\mu_3 < \mu_4$

Q.2. A comparison of 3 drugs was conducted to test whether there are any differences among their effects of relieving pain. The mean relief ratings and standard deviations are given below. Each drug was assigned at random to 10 subjects in a completely randomized design ($n_1=n_2=n_3=10$).

Drug	Mean	SD
1	8	1
2	10	2
3	12	2

H_0: $\mu_1=\mu_2=\mu_3$ H_A: Not all μ_i are equal

- Between Drug Sum of Squares and degrees of freedom: **SST = 80 dfT = 2**
- Within Drug Sum of Squares and degrees of freedom: **SSE = 81 dfE = 27**
- F-Statistic **F_{obs} = (80/2) / (81/27) = 40/3 = 13.33**
- Reject H_0 (at α=0.05 level) if F-statistic is **Above** / Below **$F_{.05,2,27}$ = 3.354**

Q.3. Ford wants to compare mean assembly times for Explorer's at their 3 assembly plants. They observe random samples of 10 cars at each plant, and obtain the following summary statistics on assembly times (in minutes):

Plant	Mean	Std. Dev.
Atlanta	**180**	**12**
Chicago	**185**	**10**
Detroit	**175**	**9**

(a) Compute the between plant (Treatment) sum of squares and its degrees of freedom **SST = 500 dfT = 2**

(b) Compute the within plant (Error) sum of squares and its degrees of freedom **SSE = 2925 dfE = 27**

(c) Compute the test statistic **F_{obs} = (250 / 2) / (2925 / 27) = 1.154**

(d) Conclude that the population means differ ($\alpha = 0.05$) if the test statistic is **> 3.354**

Q.4. A study compared the deuterium/hydrogen (D/H) ratio at methyl for 5 variety of wines grown in north Xinjiang in 2009. There were samples of 4 wines from each variety. The means and standard deviations for the D/H ratios are given below. Complete the Analysis of Variance Table, and test for difference among Variety population means.

$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$

Variety	n	Mean	SD
Merlot	4	102.63	0.22
Cabernet Sauvignon	4	101.89	0.40
Riesling	4	102.03	0.70
Chardonnay	4	101.99	0.77
Italian Riesling	4	102.11	0.26

Between Trts (SST)	1.3504
Within Trts (SSE)	4.0767

Source	df	SS	MS	F
Treatments	**4**	**1.3504**	**0.3376**	**1.242181**
Error	**15**	**4.0767**	**0.27178**	
Total	**19**	**5.4271**		

Q.5. A study was conducted, comparing 5 treatments of khat, given to female rats for sexual motivation. There were n=6 rats per treatment, and MSE = s^2 = 6.0 for the response: time spent in the incentive zone. Use Tukey's HSD and Bonferroni's method to compare all pairs of treatment means. State which pairs of treatment have significantly different means.

Groups	n	Mean	SD
A: Control	6	11.38	1.77
B: khat extract (10mg/kg)	6	9.19	2.55
C: khat extract (20mg/kg)	6	11.48	2.50
D: khat extract (50mg/kg)	6	13.64	2.38
E: khat extract microcapsules	6	21.77	2.90

Tukey's W: ______________________________ Bonferroni's B: _______________________________

$$q_{.05,5,25} = 4.153 \quad W = 4.153\sqrt{\frac{6.0}{6}} = 4.153 \qquad c = 10: \ t_{.025,10,25} = 3.078 \quad B = 3.078\sqrt{\frac{2(6.0)}{6}} = 4.353$$

E is significantly higher than all others, D is significantly higher than B

Q.6. Random samples of $n_1 = n_2 = n_3 = 12$ players each from 3 men's professional leagues were selected. The leagues were: 1=English Football (EPL), 2=North American Hockey (NHL), 3=American Basketball (NBA). Body Mass Indices were obtained for all N = 36 players sampled. Use the Kruskal-Wallis test to determine whether population medians differ among the leagues. H_0: All population medians are equal.

NBA.BMI	NHL.BMI	EPL.BMI	NBA.RANK	NHL.RANK	EPL.RANK
25.559	25.676	23.099	22	23	7
25.180	25.495	21.476	18	20	4
26.402	24.371	23.478	27	15	9
25.512	26.120	21.455	21	24	3
25.264	26.959	21.115	19	30	2
27.246	27.620	24.103	32	33	12.5
24.781	26.318	20.980	16	25	1
24.342	28.170	23.493	14	35	10
23.573	26.444	22.871	11	28	6
27.042	27.986	24.103	31	34	12.5
26.532	29.550	23.236	29	36	8
25.001	26.384	22.452	17	26	5
		Sum	257	329	80

Test Statistic: _________________ Rejection Region: _____________________________ Reject H_0? **Yes** **No**

$$H = \frac{12}{36(37)}\left[\frac{(257)^2}{12} + \frac{(329)^2}{12} + \frac{(80)^2}{12}\right] - 3(37) = 135.653 - 111 = 24.653 \quad RR: H \geq \chi^2_{2,.05} = 5.991$$

Q.7. For a given set of data for the Completely Randomized Design, which statement best describes the relation between Tukey's *W* and Bonferroni's *B*:

i) *W* will always be larger than *B* ii) ***W* will always be smaller than *B*** iii) It depends on the dataset.

Q.8. An automobile rental company is interested in comparing the mean highway mileages among 3 models of cars. They sample 10 cars of each model, and measure the mileage on each car (Y, in miles per gallon), based on a highway drive of a fixed distance. The results (mean and standard deviation) are given below. Complete the Analysis of Variance table.

Model	Mean	SD
1	17.0	5.0
2	23.0	6.0
3	20.0	4.0

ANOVA					
Source	df	SS	MS	F_obs	F(.05)
Treatments	2	180	90	3.506	3.354
Error	27	693	25.67		
Total	29	873			

Q.9. A 1-Way ANOVA model is fit comparing the weights of 4 natural fiber fabrics of common dimensions. There were 15 replicates for each of the 4 fibers (cotton, linen, silk, and wool). The sample means and error sum of squares are given below. Compute Tukey's Honest Significant Difference (*W*), and identify which fibers differ significantly.

$$\overline{Y}_C = 14.9 \quad \overline{Y}_L = 18.9 \quad \overline{Y}_S = 9.9 \quad \overline{Y}_W = 20.1 \qquad SSE = 727$$

$$q_{.05,4,56} = 3.745 \quad MSE = \frac{727}{56} = 12.982 \quad W = HSD = 3.745\sqrt{\frac{12.982}{15}} = 3.484$$

$\mu_S \quad \mu_C \quad \mu_L \quad \mu_W$

Q.10.: An experimenter is interested in comparing the mean potencies among 3 formulations of insecticides. She sets up 18 containers, each with 100 specimens. She then randomly assigns the insecticides to the containers, so that each insecticide is used in 6 containers. She measures the numbers of specimens dying for each container. The results are given below.

Formulation	#reps	Mean	SD
1	6	30	6
2	6	42	5
3	6	48	6

p.10.a. Compute the treatment (formulation) sum of squares and degrees of freedom. SST = 168 dfT=2

p.10.b. Compute the error sum of squares and degrees of freedom. SSE = 485 $df_E = 15$

p.10.c. Use Bonferroni's method to obtain simultaneous 95% Confidence Intervals for differences between population means among the 3 formulations.

$B_{ij} = 8.844$ $\mu_1-\mu_2$: (-20.844 , -3.156) $\mu_1-\mu_3$: (-26.844 , -9.156) $\mu_2-\mu_3$: (-14.844 , 2.844) p.10.d. Show your results by drawing lines that join means that are not significantly different. **1 2 3**

Q.11. A study compared life-spans among 3 species of plants in lab settings. The researchers determined that the life-spans followed distributions that were clearly not normal. They planted 10 plants from each variety, and measured the time until the plants demonstrated a particular amount of wilting. The life-spans and their ranks are given below. (Conduct the test described below at $\alpha = 0.05$ significance level)

Life-span	Species1	Species2	Species3	Rank	Species1	Species2	Species3
1	22.13	21.62	13.79	1	14	13	1
2	14.94	18.64	16.42	2	2	7	3
3	17.92	21.53	18.57	3	5	12	6
4	20.73	20.85	21.16	4	8	9	10
5	23.04	16.78	21.32	5	15	4	11
6	36.04	42.20	40.66	6	17	24	21
7	48.25	43.02	42.24	7	29	26	25
8	48.39	39.04	40.99	8	30	20	22
9	38.78	41.18	47.79	9	19	23	28
10	47.21	32.32	38.29	10	27	16	18
				Total	166	154	145

p.11.a. Compute the Kruskal-Wallis statistic for testing H_0: Species median life-spans are equal. **H = 0.286**

p.11.b. Reject / **Do not Reject** H_0 because Test statistic is Larger / **Smaller** than **5.991**

Q.12. A researcher wants to compare the effects of 3 advertisements on consumers' attitude toward a new consumer product. He obtains 60 consumers, and randomly assigns 20 to each of the advertisements (each consumer is only exposed to one of the ads). After viewing the advertisement, each person is asked to rate his/her feelings toward the product on a visual analogue scale, where higher scores are more favorable. The results (means, standard deviations, and sample sizes) are given below.

Ad	n	Mean	SD
1	20	7.0	1.5
2	20	5.5	1.6
3	20	4.0	1.4

p.12.a. Compute the Between treatment sum of squares and its corresponding degrees of freedom

Between Treatment Sum of Squares **SST = 90** Degrees of Freedom **dfT = 2**

p.12.b. Compute the Within treatment sum of squares and its corresponding degrees of freedom

Within Treatment Sum of Squares **SSE = 128.63** Degrees of Freedom **dfE = 57**

p.12.c. Test H_0: No differences among the true ad means ($\mu_1=\mu_2=\mu_3$) versus:

vs H_A: Differences exist among true ad means (Not all μ_i are equal) ($\alpha=0.05$ significance level)

p.12.c.i. Test Statistic: $\mathbf{F_{obs} = 19.941}$

p.12.c.ii. Clearly state Rejection Region: $\mathbf{F_{obs} \geq F_{.05,2,57} = 3.159}$

Q.13. A scientist is interested in comparing the duration of lifetimes of insects exposed to 4 varieties of pesticides. He runs a Completely Randomized Design with 10 insects per variety of pesticide (thus, a total of N=40). Due to some outliers, he decides to use the Kruskal-Wallis test. You are given the following partial results (Hint: 1+2+...+n = n(n+1)/2).

Pesticide (i)	Rank Sum (T_i)
1	200
2	180
3	225
4	

p.13.a. Compute $\mathbf{T_4}$ **= (40(41)/2) – (200+180+225) = 215**

p.13.b. Test H_0: No differences among the 4 varieties' medians versus H_A: Differences exist at α = 0.05 significance level.

p.13.b.i. Compute the test statistic (there were no "ties" among the data)

Test Statistic = **H = 0.841**

p.13.b.ii. Clearly state the rejection region: $\mathbf{H \geq 7.815}$

p.13.b.iii. The P-value will be (circle one) **> 0.05** < 0.05 Need More Information

Q.14. Researchers wished to compare the yields (in pounds) of 3 varieties of 4-year old orange trees in an orchard. They obtain random samples of 10 trees of each variety from the orchard and obtain the following results. Due to the skew of the distributions, they choose to use the appropriate non-parametric test.

	Yld(Lbs)			Rank	
Variety1	Variety2	Variety3	Variety1	Variety2	Variety3
0.53	0.03	1.97	6	1	13
1.24	4.86	1.07	9	26	8
2.51	5.51	0.53	16	27	5
2.08	3.75	2.91	14	22	19
5.64	0.52	1.75	28	4	11
2.15	0.91	4.46	15	7	25
0.14	1.76	3.05	2	12	21
13.42	0.42	12.08	30	3	29
2.71	2.77	1.62	17	18	10
2.95	3.86	4.31	20	23	24
		Sum	157	143	165

- Give the test statistic for testing H_0: Variety Medians all equal versus H_A: Variety Means not all equal

  H = 0.320

- We reject H_0 at the α=0.05 significance level if it falls in what range **$H \geq 5.991$**
- The P-value for this test is **greater than** / less than 0.05 (circle correct answer).

Q.15. A study is conducted to compare 2 methods of teaching foreign language to children (independent samples). One analyst uses a 2-sided test of H_0: $\mu_1 - \mu_2 = 0$ versus H_A: $\mu_1 - \mu_2 \neq 0$ based on the independent sample t-test, assuming equal variances. The other analyst uses a 1-Way Analysis of Variance F-test to test H_0: $\mu_1 = \mu_2$ versus H_A: $\mu_1 \neq \mu_2$. They use the same computing software, so there are no issues due to rounding. Choose the correct answer:

- The p-value from the t-test will always be higher than the p-value from the F-test
- The p-value from the t-test will always be lower than the p-value from the F-test
- **The p-value from the t-test will always be the same as the p-value from the F-test**
- None of the above

Q.16. A scientist wants to compare the effects of 3 treatments on behavior in mice. The treatments are:

1) Placebo, 2) Drug A, 3) Drug B. The experiment is balanced. The researcher is interested in 2 specific contrasts: Contrast 1: Placebo (μ_1) versus Average of Drug A (μ_2) and Drug B (μ_3), Contrast 2: Drug A versus B. Give the two contrasts (note there are many ways of writing these, but they share a specific pattern):

$$l_1 = +2\mu_1 - 1\mu_2 - 1\mu_3 \qquad l_2 = 0\mu_1 + 1\mu_2 - 1\mu_3$$

Q.17. For a balanced 1-Way ANOVA, with t > 2 groups, when making all pairwise comparisons, Tukey's W will always be smaller than Bonferroni's B.

TRUE or FALSE

Q.18. A 1-Way ANOVA is conducted, comparing clarity of t = 3 methods of meniscal repair. A sample of N=18 subjects was obtained and assigned at random such that n_1 = 6 received method 1, n_2 = 6 received method 2, and n_3 = 6 received method 3. The response was Y = Displacement (mm). Complete the following table to test:

$H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_A :$ Not all μ_i are equal

Source	df	SS	MS	F_obs	F(.05)
Treatment(Between)	2	105	52.5	3.768	3.682
Error(Within)	15	209	13.93	#N/A	#N/A
Total	17	314	#N/A	#N/A	#N/A

Do we reject the null hypothesis? **Yes** or No Is the P-value **< 0.05** or > 0.05

Q.19. An experiment was conducted to compare wine color intensity (Y) among t = 6 types of wine barrels. There 9 replicates for each of the wine barrel types. The Mean Square Error (MSE) was 1.04.

p.19.a. Compute Tukey's Honest Significant Difference for Comparing all pairs of wine barrel types

Conclude Conclude $\mu_i \neq \mu_{i'}$ if $\left|\overline{y}_i - \overline{y}_{i'}\right| \geq W_{ii'} = HSD_{ii'} = 4.197\sqrt{\frac{1.04}{9}} = 1.427$

p.19.b. Compute Bonferroni's Minimum Significant Difference for Comparing all pairs of wine barrel types

Conclude Conclude $\mu_i \neq \mu_{i'}$ if $\left|\overline{y}_i - \overline{y}_{ii'}\right| \geq B_{ii'} = 3.089\sqrt{\frac{2(1.04)}{9}} = 1.485$