STA 4702/5701 – Spring 2017 – Exam 2 PRINT Name ____________________

Note: Conduct all tests at at α = 0.05 significance level. SHOW ALL WORK.

Q.1. An experiment is conducted where the effects of office wall color and lighting type are observed on productivity of office workers for p=3 productivity measures. There are 4 wall colors, and 3 lighting types, and a sample of 120 workers is randomized so that 10 receive each combimation of wall color and lighting type. They are also interested in whether the wall color effects differ by lighting type and vice versa. Give the ANOVA table with all sources of variation, their degrees of freedom and the dimension of the SSCP matrix. Note that each individual worker receives only one condition.

Source	df	dimension of SSCP matrix

Q.2. An experiment is conducted to assess the effect of 2 types of kayak paddles. A group of 20 kayakists is obtained, and each kayakist uses each paddle (in random order, in the same kayak). Two responses are measured: Time to complete the course (X_1) and a subjective measurement of the comfort of the paddle (X_2). Which test is appropriate: Paired Comparison or Independent Samples? Regardless of your answer, what is the Rejection Region for the T^2 statistic for your method?

Method ________________________________ Rejection Region ____________________________________

Q.3. Quadratic growth curves are fit, relating shrinkage of fabric exposed to one of two types of treatment. For the 2 treatments, measurements of shrinkage are made at 0, 1, 2, and 3 cycles. There were 10 fabric pieces in each treatment. The fitted growth curves are. Treatment 1: $6+4t-t^2$ Treatment 2: $10+5t-2t^2$

Give the fitted growth curves at each number of cycles based on the models.

Cycles	0	1	2	3
Trt1				
Trt2				

Q.4. A study was conducted comparing attitudes of individuals toward robots for samples of n_C = 16 Chinese and n_G = 16 German subjects. The 3 responses measured were: Credibility (i=1), Likability (i=2), and Trust (i=3) toward robots. All scales were averages of responses to statements on 1-7 scales, so can be considered to be measured on comparable scales. The goal is to compare the profiles of the 2 cultures versus the responses. Means are given below.

Culture	Credibility	Likability	Trust
Chinese	4.08	4.87	4.84
German	3.24	3.57	3.95

p.4.a. We wish to test for parallel profiles for the 2 cultures:

$$H_{01}: \mu_{C2} - \mu_{C1} = \mu_{G2} - \mu_{G1} \,,\, \mu_{C3} - \mu_{C1} = \mu_{G3} - \mu_{G1} \quad \text{where: } \boldsymbol{\mu}_C = \begin{bmatrix} \mu_{C1} \\ \mu_{C2} \\ \mu_{C3} \end{bmatrix} \quad \boldsymbol{\mu}_G = \begin{bmatrix} \mu_{G1} \\ \mu_{G2} \\ \mu_{G3} \end{bmatrix} \qquad \mathbf{C_1} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$H_{01}: \mathbf{C_1}\boldsymbol{\mu}_C = \mathbf{C_1}\boldsymbol{\mu}_G \quad \Rightarrow \quad \mathbf{C_1}\left(\boldsymbol{\mu}_C - \boldsymbol{\mu}_G\right) = 0 \qquad \text{Compute } \left(\overline{\mathbf{x}}_C - \overline{\mathbf{x}}_G\right) \text{ and } \mathbf{C_1}\left(\overline{\mathbf{x}}_C - \overline{\mathbf{x}}_G\right)$$

$\left(\overline{\mathbf{x}}_C - \overline{\mathbf{x}}_G\right) =$

$\mathbf{C_1}\left(\overline{\mathbf{x}}_C - \overline{\mathbf{x}}_G\right) =$

p.4.b. $\left[\left(\frac{1}{n_C} + \frac{1}{n_G}\right)\mathbf{C_1}\mathbf{S}_{\text{pooled}}\mathbf{C_1}'\right]^{-1} = \begin{bmatrix} 619.9 & -220.6 \\ -220.6 & 590.8 \end{bmatrix}$ Compute the T^2 statistic and give the rejection region.

Test Statistic ____________________ Rejection Region ______________________ P-value > or < 0.05?

Q.5. A study compared 3 groups (normal weight, overweight, and obese) in terms of how each person estimated calories for 2 types of meals (healthy and unhealthy). Summary data are given below.

Model: $\mathbf{X}_{lj} = \boldsymbol{\mu} + \boldsymbol{\tau}_l + \mathbf{e}_{lj} \quad l = 1,...,g;\ j = 1,...,n_l \quad \mathbf{X}_{lj} = \begin{bmatrix} X_{lj1} \\ \vdots \\ X_{ljp} \end{bmatrix} \quad \boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_p \end{bmatrix} \quad \boldsymbol{\tau}_l = \begin{bmatrix} \tau_{l1} \\ \vdots \\ \tau_{lp} \end{bmatrix}$

Group	n	hmean	uhmean		B			W	
1 Normal	60	134.66	241.98		7924684	18363.68		573363.2	749868.6
2 Overweight	45	116.7	244.31		18363.68	32668.6		749868.6	2236406
3 Obese	36	96.81	218.93						
Total	141				|W|/1E10	|B+W|/1E10			
					71.99698	1869.252			

where: $\mathbf{B} = \sum_{l=1}^{g} n_l \left(\bar{\mathbf{x}}_l - \bar{\mathbf{x}}\right)\left(\bar{\mathbf{x}}_l - \bar{\mathbf{x}}\right)' \qquad \mathbf{W} = \sum_{l=1}^{g} \sum_{j=1}^{n_l} \left(\mathbf{x}_{lj} - \bar{\mathbf{x}}_l\right)\left(\mathbf{x}_{lj} - \bar{\mathbf{x}}_l\right)' \qquad \bar{\mathbf{x}}_1 = \begin{bmatrix} 134.66 \\ 241.98 \end{bmatrix}$

p.5.a. Give the following values:

$g =$ ______ $p =$ ______ $N =$ ______ $\Lambda^* =$ ____________________ $\ln\left(\Lambda^*\right) =$ ____________________

p.5.b. Conduct the exact F-test (assuming normality) for testing $H_0 : \boldsymbol{\tau}_1 = ... = \boldsymbol{\tau}_g = 0$

Test Statistic: ___________________________ Rejection Region: ______________________ p-value > or < 0.05

Continued

p.5.c. We wish to make (Bonferroni) simultaneous comparisons among all pairs of healthy means among groups, and all pairs of unhealthy means among groups. How many pairs of means are there? Give the simultaneous CI comparing healthy meal means between Normal (i=1) and Overweight (i=2) subjects.

Of comparisons ____________________ Simultaneous CI ____

Q.6. A linear regression model is fit, relating April mean Temperature (Y) to Latitude (Z_1), Longitude (Z_2), and Elevation (Z_3) to a sample of n = 50 weather stations in Texas. The partial Analysis of Variance and parameter estimates are given below. $Y = \beta_0 + \beta_1 Z_1 + \beta_2 Z_2 + \beta_3 Z_3 + \varepsilon \quad \varepsilon \sim NID\left(0, \sigma^2\right)$

ANOVA			
	df	*SS*	*MS*
Regression	3	910.4479	303.4826
Residual	46	66.93629	1.455137
Total	49	977.3842	

	Coefficients	*Standard Err*
Intercept	54.16177	21.60857
LAT	-1.48478	0.135253
LONG	0.61018	0.193845
ELEV	-0.00216	0.000531

p.6.a. Test whether Mean April temperature is associated with either Latitude, Longitude, and Elevation.

H_0: _______________________ Test Statistic: ____________________ Rejection Region: ___________________

p.6.b. Test whether, after accounting for Latitude and Elevation, Mean April temperature is associated with Longitude.

H_0: _______________________ Test Statistic: ____________________ Rejection Region: ___________________

Q.7. The anthropometric study of the Kanets of Lahoul and Kulu, measured Sitting Height (X_1), Kneeling Height (X_2), and Cubit (X_3) among random samples of $n_1 = 30$ Lahouls and $n_2 = 60$ Kanets. Summary calculations are below.

Spooled				Xbar1	Xbar2		INV((1/n1+1/n2)Sp)				INV((1/n1+1/n2)Sp)*(xb1-xb2)		
9.92	7.56	2.29		86.17	86.55		3.29	-1.51	-0.52		2.59		
7.56	15.14	3.84		120.23	122.57		-1.51	2.56	-1.93		-4.17		
2.29	3.84	3.31		44.66	45.30		-0.52	-1.93	8.64		-0.83		

p.7.a. We want to test whether the population mean vectors are the same for each of the 2 populations. H_0: $\mu_1 - \mu_2 = 0$.
Compute the Test Statistic and Rejection Region for this test.

Test Statistic: ___________________________ Rejection Region: ______________________ p-value > or < 0.05

p.7.b. We want simultaneous Confidence Intervals for differences in the population means for each of the 3 measures. Compute the Confidence Intervals based on the Simultaneous (all linear functions) and Bonferroni methods for Cubit (make adjustments so these would hold if you did this for all 3 measures).

Simultaneous CI's ______________________________ Bonferroni's Method ___________________________________