5.2

For any $\varepsilon > 0$

$$P(|\overline{X}_n - \overline{\mu}_n| > \varepsilon) \leq \frac{Var(\overline{X}_n)}{\varepsilon^2} = \frac{\sum_{i=1}^{n} \sigma_i^2}{n\varepsilon^2} \to 0 \tag{1}$$

$\Rightarrow \overline{X}_n - \overline{\mu}_n \to 0$ in probability and by condition $\overline{\mu}_n \to \mu \Rightarrow \overline{X}_n \to \mu$ in probability

5.5

Mgf of Bin(n,p) is $(pe^t + 1 - p)^n$

$(pe^t + 1 - p)^n = [1 + p(e^t - 1)]^n = [[1 + p(e^t - 1)]^{\frac{1}{p(e^t-1)}}]^{np(e^t-1)}$

$[[1 + p(e^t - 1)]^{\frac{1}{p(e^t-1)}}]^{np(e^t-1)} \to e^{\lambda(e^t-1)}$

(because $\lim[1 + a_n]^{\frac{1}{a_n}} \to e \; as \; a_n \to 0$)

5.16

$EX = 2/3 Var(X) = 1/18 ES = 40/3 Var(s) = 10/9$

$P(S \leq 10) = P(\frac{S-ES}{\sqrt{Var(S)}} \leq \frac{10-40/3}{\sqrt{10/9}}) \approx \Phi(-\sqrt{10}) \approx 0.0008$

5.26

$S = \sum_{i=1}^{25} X_i$; $X_i \sim Bin(1, 0.3)$; Xi are i.i.d

$P(S \leq 5) \approx \Phi(-1.09) \approx 0.1379; P(S \leq 7) \approx \Phi(-0.218) \approx 0.4129;$

$P(S \leq 9) \approx \Phi(0.655) \approx 0.7454; P(S \leq 11) \approx \Phi(1.528) \approx 0.937$

6.5

$X \sim F_{n,m}$ From definition $W = \frac{U/n}{V/m} \sim F_{n,m}; U \sim \chi_n^2; V \sim \chi_m^2;$ U, V are independent. W^{-1} has the same distribution of X^{-1} and $W^{-1} = \frac{V/m}{U/n} \sim F_{m,n}$. so $X^{-1} \sim F_{m,n}$.

6.6

$T \sim t_n$. From definition $X = \frac{Z}{U/n} \sim F_{1,n}; Z \sim N(0,1); U \sim \chi^2_n$; Z, U are independent. X^2 has the same distribution of T^2 and $X^2 = \frac{Z^2/1}{U/n} \sim F_{1,n}$. (because $Z^2 \sim \chi^2_1; U \sim \chi^2_n$; Z^2, U are independent.) so $X^{-1} \sim F_{m,n}$.

6.10

$P(a < S^2/\sigma^2 < b) = P((n-1)a < (n-1)S^2/\sigma^2 < (n-1)b) = P((n-1)a < \chi^2_{n-1} < (n-1)b)$ (Because $(n-1)S^2/\sigma^2 \sim \chi^2_{n-1}$)

6.11

$(n-1)S_X^2/\sigma^2 \sim \chi^2_{n-1}; (n-1)S_Y^2/\sigma^2 \sim \chi^2_{m-1}$ and S_X^2, S_Y^2 are independent. So $\frac{S_X^2}{S_Y^2} = \frac{(n-1)S_X^2/(n-1)\sigma^2}{(m-1)S_X^2/(m-1)\sigma^2} \sim F_{n-1,m-1}$