

a: simple hypothesis ; C: composite hypothesis.

b: simple d: composite

$$(a) \Lambda_1 = \frac{f_0(x_1)}{f_A(x_1)} = \frac{0.2}{0.1} = 2 ; \Lambda_2 = \frac{f_0(x_2)}{f_A(x_2)} = \frac{0.3}{0.4} = 0.75 ; \Lambda_3 = \frac{f_0(x_3)}{f_A(x_3)} = \frac{0.3}{0.1} = 3$$
$$\Lambda_4 = \frac{f_0(x_4)}{f_A(x_4)} = \frac{0.2}{0.4} = 0.5 ; \quad \Lambda_4 < \Lambda_2 < \Lambda_1 < \Lambda_3$$

(b)  $\alpha = 0.2$

$$P\left(\frac{f_0(x)}{f_A(x)} \leq 0.5\right) = P(X=x_4) = 0.2$$

$\therefore$  when  $X=x_4$  we reject  $H_0$

$\alpha = 0.2$

$$P\left(\frac{f_0(x)}{f_A(x)} \leq 0.75\right) = P(X=x_4 \text{ or } X=x_2) = 0.2 + 0.3 = 0.5$$

$\therefore$  when  $X=x_4$  or  $X=x_2$  we reject  $H_0$

$$\Lambda = \frac{f(x_1, \dots, x_n | \theta_0)}{f(x_1, \dots, x_n | \theta_1)} = \frac{g(T(x_1, \dots, x_n), \theta_0) h(x_1, \dots, x_n)}{g(T(x_1, \dots, x_n), \theta_1) h(x_1, \dots, x_n)}$$

$$= \frac{g(T(x_1, \dots, x_n), \theta_0)}{g(T(x_1, \dots, x_n), \theta_1)} = A(T) \text{ this is a function of } T$$

if the distribution  $T$  is known, we can use this distribution to find rejection region:  $\{A < C\}$

$$\alpha = P\{A < C\} = P\{A(T) < C\}$$